3D Modelling of Pressure Exerted by Polymeric Knittings with Increased Elasticity on Cylindrical Surfaces

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The elastic contention is the most efficient and used method to solve chronic venous insufficiency, being used in some countries by 71% of patients. In functional design of knitted orthoses with increased elasticity which exert a controlled pressure on a cylindrical surface of human body, used in the treatment of venous insufficiency, the model proposed allows optimal assessment of pressure values in knitting - anthropometric sizes - applied pressure ratio. Therefore starting from mechanical behaviour described by stress-strain diagrams of knittings from polyurethane yarns and using the pressure formula based on the model proposed we can assess accurately the applied pressure values for different anthropometric circumferences. Based on the model it was determined the calculation ratio of pressure exerted by the knitting versions for the cylindrical model embracement of different parts of human body. The choosing of orthoses is a difficult problem, few patients can find serial orthoses which fit the patients' limbs configuration. Hence, the model can be used easily for designing and manufacturing on demand of the orthoses with controlled pressure potential.

Keywords: graduated compression stockings, pressure modelling, stress-strain diagrams, chronic venous insufficiency, knitted fabrics, polyurethane yarns

The medical textile materials interact with a part of body surface exerting a certain pressure with the effect of tissue constriction resulting that blood vessels are narrowing regardless of their section dimension. The pressure determines, according to Bernoulli's principle [1], an increase of blood speed through circulatory system vessels causing at the same time an acceleration of biological interactions starting from cellular, capillary level, etc, with the revascularization of implied area, a better blood irrigation, oxygenation and tissue feeding. The materials can be applied as bandages, knitted orthoses for patients with chronic venous insufficiency of the leg and others [2-9]. The property of knitting from polyurethane yarns of certain pressure on a surface depicted of human body is described from a mechanical point of view by a multitude of indicators determined in statical or dynamic conditions: elasticity or plasticity, tensile and stretching resistance [7-9]. Through mechanical-rheological indicators analysis in the ratio: yarn – technological parameters of knitting – knitting's – exerted pressure there were established the values of elastic force for which the length deformations modifications of materials knitted from polyurethane yarns which take place in time, could fit in the elastic deformations domain [7-22]. The researches performed for the aim of determining the values of pressure located by knitted materials with increased elasticity on a cylindrical surface of human body are presented below.

Experimental part

Mathematical model for calculating pressure exerted of materials knitted from elastomeric yarns on cylindrical surfaces

It is known that medical stockings or ortheses with compression potential exercise a gradual pressure on the foot, the pressure values being established according to disease. We proposed to determine , in the first stage of mathematical model, the values of force exercised on the foot by a knit variant with the following structure : Raw material – Elastomeric Core Yarn156 dtex ; Knit structures -single jersey with missed stitches. The investigated knittings have been made on Lonati small circular knitting machines, fineness 32 E. The knit variant was tested on the electronic tensile strength tester Tinius Olsen, according to ISO 1421 resulting the stress-strain diagram (fig.1). The scientific process of exercised force analysis on the foot, considered as a cylindrical surface, according to deformation, is presented in figure 2. Figure 2 presents the model of aproximation of curve described by stress-strain diagram.

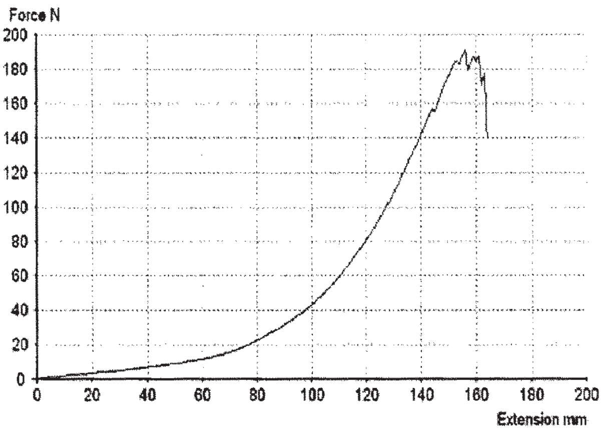

Fig.1. Stress-strain diagram for knit version

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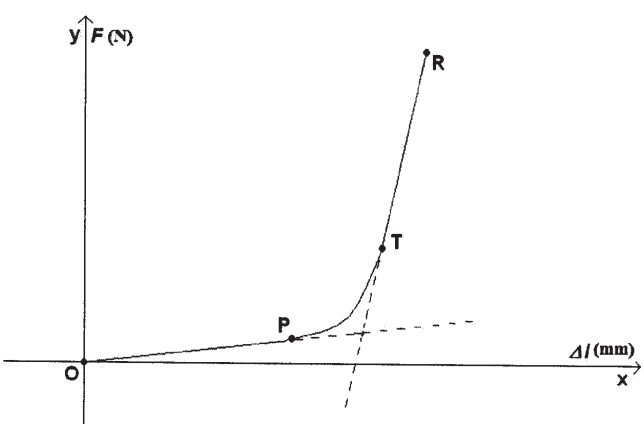

Fig.2. The proposed model: the rate of stress-strain diagrams

We notice on the proposed model diagram 3 portions of the curve, thus:

a. the OP area, of Hooke proportionality, as a straight line segment which starts from the origin of the axes system, with inclination determined by the point coordinates P(x1,y1);

b. the PT area, Hooke's law does not apply, and the elastic force is not anymore proportional with the relative strain;

c. the TR area before breaking, being a segment of straight line, with inclination determined by the points coordinates $T(x_2,y_2)$ and $R(x_3,y_3)$. Also on this part Hooke's law does not apply, because the segment straight line does not go through the origin of the axes system.

Results and discussions

The parametrization of this model for various practical experiments lies in:

- the determination as precise as possible of points' coordinates $P(x_1,y_1)$, $T(x_2,y_2)$ and $R(x_3,y_3)$. These will lead to the parametrization of the 2 straight line segments;
- the synthesis of a interpolation function for PT area, which should represent as well the experimental model on this portion. Thus we can notice the ratio between: behaviour and function approximation by the chosen interpolation method and the complexity of the resulted mathematical model.

The establishment of the function described by the stress-strain diagram for knits with increased elasticity

The 2 portions on the diagram, characterized by straight line segments, those being OP area and TR area, can be defined according to figure 2 with:

The OP area:

On this interval, the elastic force is:

$$F_{e|OP} = m_1 \cdot \Delta l \qquad (1)$$

where:

Fe = elastic force(N),

m_1=inclination determined by point P;

Δl = the absolute elongation of the knit's version (mm);

The point P(x1,y1) determines the inclination:

$$m_1 = y_1 / x_1;\ m_1 > 0: \qquad (2)$$

The TR area:

For this interval, the elastic force is :

$$F_1 | m_2\Delta l + n_2 \qquad (3)$$

The points $T(x_2,y_2)$ and $R(x_3,y_3)$, according to figure 2 determine the straight line inclination and free term thus :

$$m_2 = y_3 - y_2 / x_3 - x_2;\ m_2 > 0; \qquad (4)$$

$$n_2 = x_3 \cdot y_2 - x_2 \cdot y_3 / x_3 - x_2;\ n_2 < 0 \qquad (5)$$

The PT area:

For the determination of the PT area we used Hermite polynomial interpolation through 2 points, complying with the differentiability condition only in the first point. Therefore, we proposed to obtain a grade 2 polynom, which should guarantee the convexity condition on the whole PT interval. Thus, the polynom becomes:

$$H(x) = c_2x^2 + c_1x + c_0, \text{ with } c_i \in \Re,\ i = 0..2 \qquad (6)$$

and are obtained: $H'(x) = 2c_2x + c_1$, $H''(x) = 2c_2$ (7)

The convexity conditions implies: $H''(x) > 0 \Rightarrow c_2 > 0$

The system is reduced to linear equations:

$$\begin{cases} H(x_1) = y_1 \\ H(x_2) = y_2 \\ H'(x_1) = m_1 \end{cases}, \text{ with } A = \begin{pmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ 2x_1 & 1 & 0 \end{pmatrix},$$

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ m_1 \end{pmatrix}, \text{ where: } m_1 = y_1 / x_1 \text{ is slope of } (d1)$$

Thus it results: $\Delta = \det(A) = (x_2 - x_1)^2 \Rightarrow \Delta > 0$ and coefficients:

$$c_2 = (y_2 - y_1) - m_1(x_2 - x_1) / (x_2 - x_1)^2, \qquad (8)$$

$$c_1 = m_1(x_2^2 - x_1^2) - 2x_1(y_2 - y_1) / (x_2 - x_1)^2, \qquad (9)$$

$$c_0 = c_2x_1^2 \qquad (10)$$

We can see $c_2 > 0$, thus the convexity condition is accomplished

The geometric interpretation for the positive sign of the coefficient

According to the formula (8) for coefficient, the sign is highlighted by the figure 3.

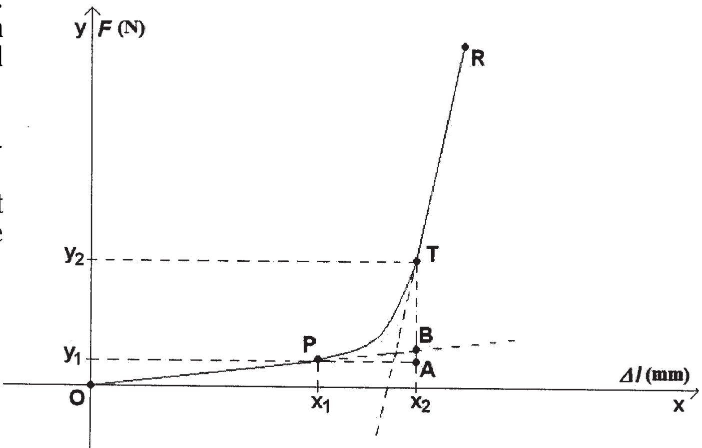

Fig.3. The geometric interpretation for the positive sign of c_2 coefficient

The coefficient numerator represents: TA-BA=TB. The positive sign is guaranteed by the more than linear increase of the function on PT inclination. Thus the force expression for the 3 areas from stress-strain diagram according to the absolute elongation at breaking Δl becomes:

$$f(x): \begin{cases} y = F \\ x = \Delta l \\ y = f(x) \end{cases} \Rightarrow F = f(\Delta l) = \begin{cases} m_1 \cdot \Delta l & , 0 \le \Delta l < \Delta l_P \\ H(\Delta l) & , \Delta l_P \le \Delta l < \Delta l_R \\ m_2 \cdot \Delta l + n_2 & , \Delta l_R \le \Delta l < \Delta l_T \end{cases} \qquad (11)$$

where F=force [N], Δl =absolute elongation (mm); the points $P(x_1,y_1)$, $T(x_2,y_2)$, $R(x_3,y_3)$ are determined on the stress-strain diagrams of the investigated sample; the inclination of d_1 and d_2 straight lines, respectively m_1 and m_2, the free term n_2 and the polynome $H(x)$ are defined by the formulae (2, 4 - 6).

Through the above analysis, one can conclude:

$0 < x_1 < x_2 < x_3$

$0 < y_1 < y_2 < y_3$

f: $[0, x_3) \rightarrow [0, y_3)$, the function is convex and increasing and

$$f': (0,x_2) \cup (x_2,x_3) \rightarrow (0,y_3),$$

$$f'(\Delta l) = \begin{cases} m_1 & , 0 < \Delta l < \Delta l_P \\ 2 \cdot c_2 \cdot \Delta l + c_1 & , \Delta l_P \le \Delta l < \Delta l_R \\ m_2 & , \Delta l_R < \Delta l < \Delta l_T \end{cases} \quad (12)$$

$$f'': (0,x_1) \cup (x_1,x_2) \cup (x_2,x_3) \rightarrow (0,y_3),$$

$$f''(\Delta l) = \begin{cases} 0 & , 0 < \Delta l < \Delta l_P \\ 2 \cdot c_2 & , \Delta l_P < \Delta l < \Delta l_R \\ 0 & , \Delta l_R < \Delta l < \Delta l_T \end{cases} \quad (13)$$

For the ratio generalization, it should be parametrized according to relative elongation thus:

$$\varepsilon_r = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0} \Rightarrow \Delta l = \varepsilon_r \cdot l_0 \quad (14)$$

where l0 is the tested sample lenght (mm), Δl = absolute elongation (mm).

Parametrized like this, the force ration according to elongation becomes:

$$F = f(\varepsilon_r) = \begin{cases} m_1 \cdot \varepsilon_r \cdot l_0 & , 0 \le \varepsilon_r < \varepsilon_{rP} \\ H(\varepsilon_r \cdot l_0) & , \varepsilon_{rP} \le \varepsilon_r < \varepsilon_{rR} \\ m_2 \cdot \varepsilon_r \cdot l_0 + n_2 & , \varepsilon_{rR} \le \varepsilon_r < \varepsilon_{rT} \end{cases} \quad (15)$$

where:

$$\varepsilon_{rP} = x_1/l_0, \quad \varepsilon_{rT} = x_2/l_0, \quad \varepsilon_{rR} = x_3/l_0, \quad (16)$$

m_1, m_2 and n_2 are defined by relations (2, 4, 5)

and $H(\varepsilon_r \cdot l_0) = c_2(\varepsilon_r \cdot l_0)^2 + c_1(\varepsilon_r \cdot l_0) + c_0$,

with $c_i \in \Re$, $i = 0..2$ and $c_2 > 0$

$\Rightarrow H(\varepsilon_r \cdot l_0) = (c_2 \cdot l_0^2) \cdot \varepsilon_r^2 + (c_1 \cdot l_0) \cdot \varepsilon_r + c_0$

We designate

- $K(\varepsilon_r) = k_2 \cdot \varepsilon_r^2 + k_1 \cdot \varepsilon_r + k_0$, (16)

  with: $k_2 = c_2 \cdot l_0^2; k_1 = c_1 \cdot l_0; k_0 = c_0$
- $M_1 = m_1 \cdot l_0; M_2 = m_2 \cdot l_0; N_2 = n_2$ (17)

And the parametrized ratio according to relative elongation becomes:

$$F = f(\varepsilon_r) = \begin{cases} M_1 \cdot \varepsilon_r & , 0 \le \varepsilon_r < \varepsilon_{rP} \\ K(\varepsilon_r) & , \varepsilon_{rP} \le \varepsilon_r < \varepsilon_{rR} \\ M_2 \cdot \varepsilon_r + N_2 & , \varepsilon_{rR} \le \varepsilon_r < \varepsilon_{rT} \end{cases} \quad (19)$$

The establishment of the calculus ratio of exercised pressure on the knit versions for embracing a cylindrical model of different areas of human body

For the pressure calculus, we considered that the knit version in extended state will coil up around a cylindrical body, establishing this way the pressure value (fig.4).

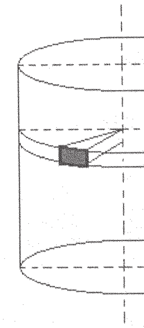

Fig.4. The cylindrical model for various parts of human body

A tension acts on the surface unit on parallel circles, which compensates with the force, which will determine pressure. The particularization of the forces course can be seen in figure 5.

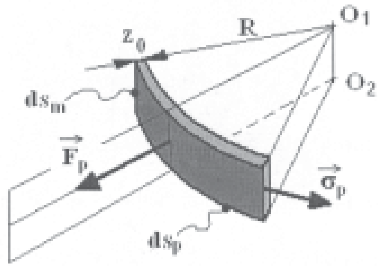

Fig.5. The representation of forces which act on surface unit (3D view)

We have:

$$F_p = p \cdot S_{unit} = p \cdot ds_m \cdot ds_p, \quad (20)$$

where:

ds_m = meridian element;

ds_p =parallel circle element.

The same forces, on parallel circles plane can be seen as in figure 6.

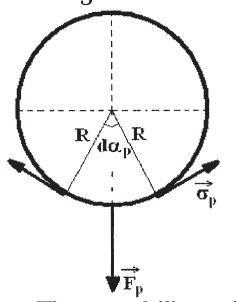

Fig.6. The representation of the forces which act on surface unit (2D view)

The equability ratio can be expressed:

$$F_p = 2 \cdot \sigma_p \cdot z_0 \cdot ds_m \cdot sin(d\alpha_p/2) \quad (21)$$

For the surface element small enough, the parallel circle element is, also infinitesimal, for which the angle at center of the circle is almost null and we can consider that:

$$sin(d\alpha_p/2) = d\alpha_p/2 \quad (22)$$

Besides that, the ratio between the arc of circle and its radius is:

$$ds_p = R \cdot d\alpha_p, \text{where } R = L/2\pi \quad (23)$$

Elastic force, is related with the tension parallel circle following relationship:

$$\sigma_p \cdot z_0 \cdot ds_m = (F_e)_{ds_m} = F_e \cdot ds_m/h_0 \quad (24)$$

The above ratios lead to the formula:

$$p = \pi \cdot F_e/h_0 \cdot L \quad (25)$$

where:

Fe= the elastic force exercised on the parallel circle;

L=the circle girth;

h0= the altitude of knit version;

$P(x_1, y_1)$		$T(x_2, y_2)$		$R(x_3, y_3)$	
x_1	y_1	x_2	y_2	x_3	y_3
50	10	135	100.00	158	145

Table 1
VALUES OF POINTS FROM STRESS-STRAIN DIAGRAM (FIG.1)

m_1	$H(x) = c_2x^2 + c_1x + c_0$			m_2	n_2
	c_2	c_1	c_0		
0,200	0.010	-0,810	25,26	1,957	-164.13

Table 2
SYNTHESIS OF FORCE FUNCTION OF ABSOLUTE ELONGATION ΔL ACCORDING TO FORMULA (2), (4), (5), (6), (8), (9), (10)

M_1	$K(x) = k_2x^2 + k_1x + k_0$			M_2	N_2
	k_2	k_1	k_0		
12	36,37	-48,62	25,26	117,39	-674

Table 3
ESTABLISHMENT OF FUNCTION THROUGH PARAMETRIZATION WITH RELATIVE ELONGATION USING THE RATIOS (16), (17), (18)

h= the altitude of cylindrical area where the material was applied.

In the above model, we did not consider the longitudinal elongation of the knitted material (along the meridian). If we also consider the longitudinal elongation, we can see that, especially at the cylindrical model, the meridional elastic force does not interfere with the pressure formula, but only the meridional relative elongation. So, if we consider the meridional relative elongation:

$$(\varepsilon_r)_m = h - h_0/h_0, \qquad (26)$$

The formula of pressure becomes :

$$p = p(\varepsilon_r) = \pi \cdot F_e(\varepsilon_r)/h_0 \cdot L \cdot ((\varepsilon_r)_m + 1)[Pa] \qquad (27)$$

The application of this mathematical model to investigated knit versions

The knit version has been tested on electronic tensile strength tester Tinius Olsen, the strain being done on transversal, the sample has 50 mm width and 60mm length (l_0 = 60mm; h_0 = 50mm). On table 1 are presented the values of P, T and R points coordinates from stress-strain diagram presented in figure 1.

Through parametrization with relative elongation according to ratio(19), the function becomes:

$$F = f(\varepsilon_r) = \begin{cases} 12 \cdot \varepsilon_r & , 0 \le \varepsilon_r < 0.83 \\ 36.37 \cdot \varepsilon_r^2 - 48.62 \cdot \varepsilon_r + 25.26 & , 0.83 \le \varepsilon_r < 2.25 \\ 117{,}39 \cdot \varepsilon_r - 674 & , 2.25 \le \varepsilon_r < 2.63 \end{cases} \qquad (28)$$

Where: ε_{rP}, ε_{rT}, ε_{rR} have been determined according to ratio (16), (To =60mm). For the on view knit version the relative elongation has been calculated between the material girth and the circumference of the leg area where the material will be applied. We considered a leg area as a cylindrical surface with girth of 350mm and height h = 90mm, knit version circumference: 194mm, leg area circumference: 350mm (L = 350mm):

$$(\varepsilon_r) = 350 - 194/194 = 0{,}8 \qquad (29)$$

The relative elongation calculated according to leg circumference and stocking circumference (ε_r = 0.8) places us in the OP portion of the diagram (first interval in function expression), the value of exercised force is :

$$F = M_1 \cdot \varepsilon_r = 12 \cdot 0.80 = 9.6[N] \qquad (30)$$

For the knit version the meridional relative elongation $(\varepsilon_r)_m$ according to ratio (26) is:

$$(\varepsilon_r)_m = 0{,}8 \qquad (31)$$

The pressure exercised by knit version according to formula (27), for a leg area with a 350mm circumference and height 90mm is:

$$p = \pi \cdot 9.6/0.05 \cdot 0.35 \cdot (0.8 + 1) = \\ = 957.26[Pa] \Leftrightarrow p = 7.18[mmHg] \qquad (32)$$

On the functional design of knitting with increased elasticity, which should exert a controlled pressure on a human body area according to medical impairment, the application of the above mathematical model should allow the optimal establishment of pressure values for the ratio knitting – anthropometric dimensions – exerted pressure. This model can be applied for other knitting, with other compositions/ knitted structures, which have a similar behaviour at pulling according with figure 1.

Conclusions

The proposed model establishes the values of exerted pressure by polyurethane yarns in knitted materials with a normal behavior to pulling according to an elastic body. In the manufacturing technological processes of knitted medical products which are used in treatments based on compression for venous system disorders, the proposed model allows the establishment as accurate as possible the values of exerted pressure according to product parameters namely the transversal and longitudinal relative elongation, the value of force according to the leg circumference or any other cylindrically shaped human body area contributing to the development of functional design of many products. Further for the same version of knitting, one can determine the exerted pressure value for other circumferences of human body, establishing this way the ratio between functional design in product parameters – anthropometric dimensions – applied pressure.

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