Determination of Damping Factor at the Vibrations of Composite Bars Reinforced with Carbon and Kevlar Texture

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In this paper, it is showed that a mechanical model with one degree of freedom can be used to study the vibrations of a bar in a certain point. The considered structures were made of epoxy resin reinforced with carbon and carbon-kevlar texture. The vibrations in the free end of the bar, clamped at the other end, were determined experimentally for four different lengths. Eigenfrequencies and damping factor for the first modes of vibration were obtained for each bar. Relationships for the eigenfrequency and damping factor are proposed, as a function of the length of the bar, for each of the studied materials.

Keywords: Eigenfrequency, damping factor, composite bar

Classical theories for isotropic plates (Classical Lamination Theory - CLT) based on Poisson – Kirchhoff hypotheses, are inadequate for predicting the behaviour of anisotropic plates [1]. Furthermore, the Reissner – Mindlin type models (First Shear Deformation Theory – FSDT) can lead to large errors in the prediction of local response of moderately thick laminated plates. Discontinuous derivates in correspondence to each interface, which cannot be described by the CLT or FSDT analyses, are emphasized in [2].

A much better description can be obtained by the use of layer-wise models (LWM). The first attempts to consider each layer in a sandwich structures as a separate bar were made in [3, 4].

In [5] was applied the abovementioned assumption to laminated plates by considering each layer as an individual Reissner – Mindlin plate. In [6] and [7], each layer is treated separately using linear in – plane displacements in the thickness direction *Oz*. 8.Other researchers [8] treated each layer separately, by employing a higher – order displacement field for flexural wave propagation analysis in laminated plates.

A Reissner mixed variational equation is employed in [9] to derive the differential equations, in terms of the introduced stresses and displacement variables, that give the dynamic equilibrium and compatibility of each layer. A numerical investigation has been conducted for free vibration response of cross – ply laminated, simply supported, thick and thin plates for which closed - form solutions are given.

Studies on the damped vibrations of Euler - Bernoulli and Timoshenko beams were also undertaken, [10, 11]. The material was assumed to be incompressible whereby the same viscoelastic operators could be used both for the flexural and shear deformations. This allowed the use of the normal modes and their orthogonality conditions to solve this viscoelastic forced vibration problem. A damped and axially loaded Timoshenko beam for random transverse load is analyzed in [10]. Only a special case of damping in the transverse and rotation motion was considered, allowing then the use of orthogonality conditions of the undamped modes to decouple the modal equations. The “closed – form” solution, but for an incomplete differential equation of a simply – supported bar with external damping was obtained [11].

In [12] is presented a general modal approach to solve the linear vibration problem of a uniform Timoshenko beam with external transverse and rotation viscous damping and different viscoelastic damping in the flexural and shear deformations. With this approach, a beam with given boundary conditions can be as conveniently analyzed as a simply supported beam.

The dynamic response of a general class of continuous linear vibrating systems, which passes damping properties close to those resulting in classical (uncoupled) normal modes was analyzed [13]. First, conditions are given for the existence of classical modes of vibrations in a continuous linear system, with special attention being paid to the boundary conditions. Regular perturbation expansions in terms of undamped mode shapes are utilized to analyze the eigenproblem as well as the vibration response of almost classically damped systems. The analysis is based on a proper splitting of the damping operators in both the field equations and the boundary conditions.

Theoretical considerations

Equations of motion for vibration of composite bars are determined based on the following assumptions:

- on the outside surface, there are no distributed forces and couples;
- during movement, additional supports or other connections which involve appearance of shocks do not appear;
- the initial status of the bar is considered undeformed, so the bar free of stresses is straight;
- a plane section, normal on the axis of the bar before deformation, remains plane and normal to the axis of the bar after deformation (Bernoulli’s hypothesis).

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An important simplification of the equations of motion is obtained for the bar of constant cross section, with two axes of geometric symmetry (mass and elastic), made of orthotropic materials. In this case, the mathematical model of transversal vibrations of a composite bar is, [14, 15]:

$$\begin{aligned} &\langle \rho I \rangle \ddot{W} + c\langle \rho A \rangle \ddot{W} - K\langle AG \rangle \left[\frac{\partial^2 W}{\partial x^2} - \frac{\partial \theta}{\partial x} \right] = p, \\ &\langle \rho I \rangle \frac{\partial \ddot{\theta}}{\partial x} - K\langle AG \rangle \left[\frac{\partial^2 W}{\partial x^2} - \frac{\partial \theta}{\partial x} \right] - \langle EI \rangle \frac{\partial^3 \theta}{\partial x^3} = 0, \end{aligned} \quad (1)$$

where:

- *W(x,t)* is the transversal displacement of the elastic center of the cross-section;
- θ *(x,t)* is the rotation of the cross-section;

$$- \langle \rho A \rangle = \iint\limits_{(s)} \rho \, dS \, ; \, \langle EA \rangle = \iint\limits_{(s)} E \, dS \, ; \, \langle EI \rangle =$$

$$= \iint\limits_{(s)} E y^2 dS \, ; \, \langle AG \rangle = \iint\limits_{(s)} G \, dS \, ; \, \langle \rho I \rangle = \iint\limits_{(s)} \rho \, y^2 dS \, ; \quad (2)$$

- ρ*(x,y)* is the mass density of the materials;
- *E(x,y)* is the Young's modulus of the materials;
- *G(x,y)* is the shear modulus of the materials;
- *p(x,t)* is the transversal loading applied on the bar;
- *c* is the damping coefficient per unit of mass of the material;
- *K* is a coefficient that takes into account the non-uniformity of the shear stresses.

In the case of a thin bar (Euler-Bernoulli), one can consider that the cross section remains plane and perpendicular to the neutral axis during deformation. In this case, the equation for transversal vibrations of the bar is:

$$\langle \rho A \rangle \ddot{W} + c\langle \rho A \rangle \dot{W} + \langle EI \rangle \frac{\partial^4 W}{\partial x^4} = p. \quad (3)$$

The bar vibration is under the form:

$$W(x,t) = \sum_{n=0}^{\infty} e^{-\mu_n t} \left(A_n(x) \sin \omega_n t + B_n(x) \cos \omega_n t \right). \quad (4)$$

Applying the Laplace transform in equation (4), one obtains:

$$\overline{W}(x,s) = \sum_{n=0}^{\infty} \frac{\omega_n A_n(x) + (s + \mu_n) B_n(x)}{(s + \mu_n)^2 + \omega_n^2}, \quad (5)$$

where $(\omega_n)_{n \in N}$ is the set of eigenpulsations, and $(\mu_n)_{n \in N}$ is the set of damping factors.

Relationship (5) shows that, in order to study the vibration in a section of the bar with abscissa $x = x_0$, one may use a mechanical model with one degree of freedom, whose equation of motion in Laplace image is, [16]:

$$\overline{W}(x_0, s) = \frac{s \cdot W(x_0, 0) + \dot{W}(x_0, 0)}{m \cdot s^2 + E(s)}, \quad (6)$$

where *m* is the model mass, and *E(s)* is the bar characteristic that depends on:

- the boundary conditions, which depend on the type of supports;
- the elastic properties of the composite material;
- the damping coefficient;
- the transversal dimensions of the bar section;
- the length of the bar.

The characteristic *E(s)*, which models the bar, is chosen in such a way in which relationships (5) and (6) coincide for the section $x = x_0$, in which the vibrations are studied.

For the first mode of vibration, the characteristic *E(s)* has the form:

$$E(s) = Cms + 4\pi^2 m \frac{\langle EI \rangle}{\langle \rho A \rangle} \cdot \frac{\alpha^2}{l^3}, \quad (7)$$

where α is a coefficient that depends on how the mass of the bar is distributed.

For a clamped bar of length *l*, if one considers the same damping for its eigenmodes, then the functions $A_n(x)$ and $B_n(x)$ are:

$$\begin{aligned} A_n(x) &= C_n \left[ch\beta_n x - \cos \beta_n x - \frac{\cos \beta_n l + ch\beta_n l}{sh\beta_n l + \sin \beta_n l} (sh\beta_n x - \sin \beta_n x) \right], \\ B_n(x) &= C_n \cdot \frac{\omega_n}{\mu} \cdot \left[ch\beta_n x - \cos \beta_n x - \frac{\cos \beta_n l + ch\beta_n l}{\sin \beta_n l + sh\beta_n l} (sh\beta_n x - \sin \beta_n x) \right], \end{aligned} \quad (8)$$

where the constants C_n depend on the initial deformation of the bar, μ = c/2 is the damping factor, and

$$\beta_n = \sqrt[4]{\frac{\langle \rho A \rangle}{\langle EI \rangle} \left(\mu^2 + \omega_n^2 \right)}, \quad (9)$$

is the set of solutions for the equation, [17-18]:

$$ch\beta l \cdot \cos \beta l + 1 = 0. \quad (10)$$

In the case of a model with one degree of freedom, the following equation is obtained for the first eigenfrequency:

$$\upsilon = h \sqrt{\frac{E}{\rho}} \cdot \frac{\alpha}{l^{1,5}}, \quad (11)$$

where:

- *h* is the thickness of the bar;
- *E* is the Young's modulus of the material;
- ρ is the mass density of the material;
- α is a coefficient that depends on how the mass of the bar is distributed.

Experimental part

Three sets of test samples made of composite materials, based on epoxy resin were manufactured. For the first two sets, carbon-kevlar texture reinforcement was considered, the difference between them being given by the direction of application (fig. 1 and 2). The third set of samples has carbon texture reinforcement (fig. 3).

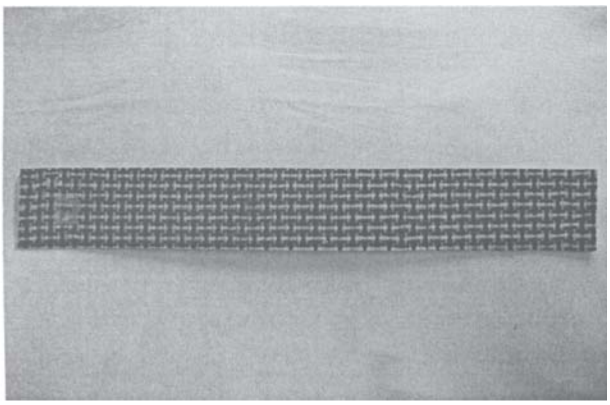

Fig. 1.

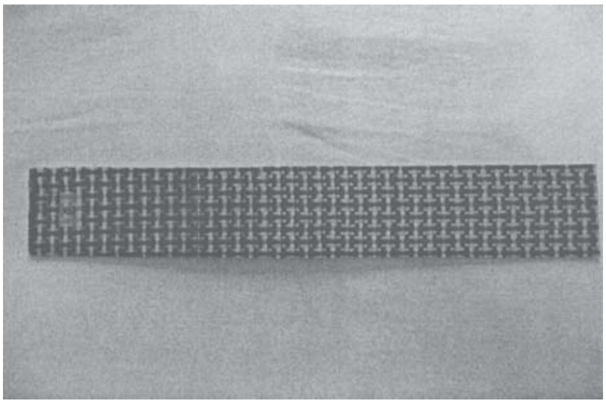

Fig. 2.

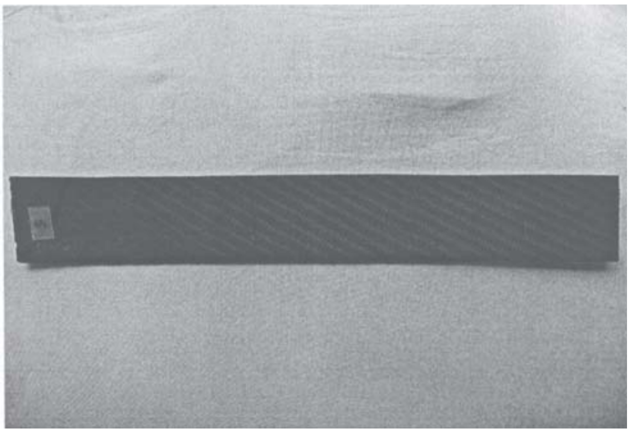

Fig. 3.

The test-samples were loaded in traction. For each set of samples, three with the highest tensile strength were selected. In table 1, the obtained mechanical properties, as the average of the results obtained for the three samples, are listed.

Free-vibrations for a beam made of each set of materials were studied in order to determine the damping factor. The experimental set-up consists of a Spider 8 data acquisition system and NEXUS 2692-A-0I4 signal conditioners (fig. 4).

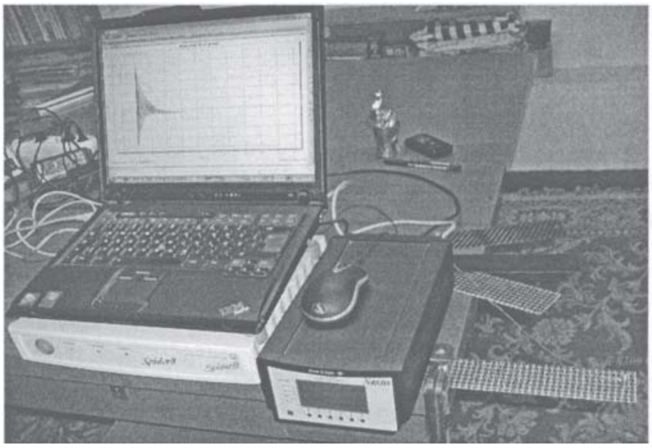

Fig. 4 Experimental set-up for determination of damped free vibrations

The beams were clamped on a massive table using a vise. The free length of each beam was: 95, 125, 155 and 185 mm. An accelerometer B&K 8309 was glued with wax on the longitudinal axis at the free end of the beam. For each condition of mounting, type of beam and free length, an initial deformation was imposed. When the beam was released, free damped vibrations occurred, which were recorded for about five seconds with a sampling frequency of 2400 Hz/ channel.

A "pass up" filtration with a lower frequency of 3 Hz was made, in order to eliminate a possible off-set introduced by the equipment. the method of logarithmic decrement was used to determine the damping of the system. Using cursors, two local maximum values X_2 and X_1 in the interval of time *Dt* were obtained. The damping factor was calculated with the relationship:

$$\mu = \frac{1}{\Delta T} \cdot \ln\left(\frac{X_2}{X_1}\right). \qquad (12)$$

Then, the damping curve f(t) = X_0 . exp(-μ . t) was plotted and superimposed on the recorded characteristics, which overlaps on the recorded characteristics. Here, X_0

Table 1

Material	Mass density *(kg/m³)*	Young's modulus *(MPa)*	Ultimate tensile strength *(MPa)*	Ultimate elongation (%)
1	1090	22685	276	1.58
2	1170	21928	286	2.30
3	1260	24944	302	1.90

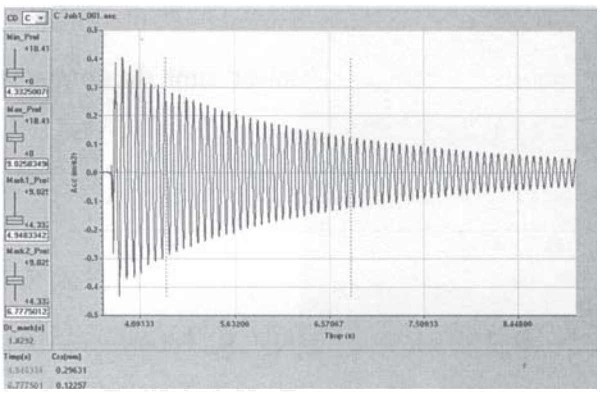

Fig. 5. Damped free vibrations for beam 1

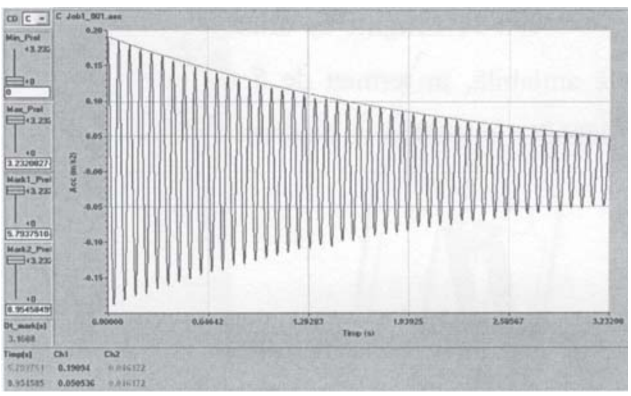

Fig. 6 Determination of damping factor for beam 1

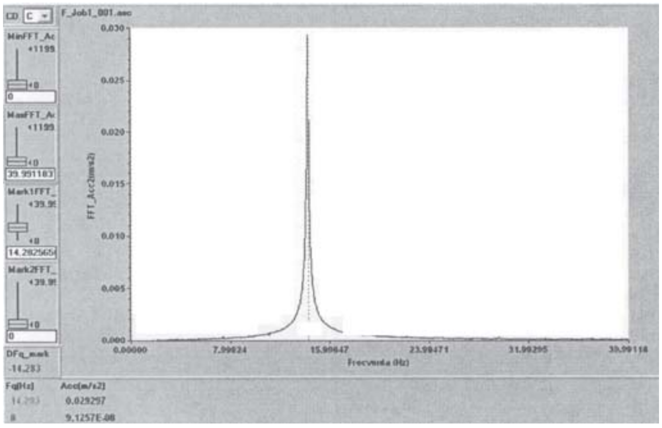

Fig. 7 Spectral analysis for damped free vibration for beam 1

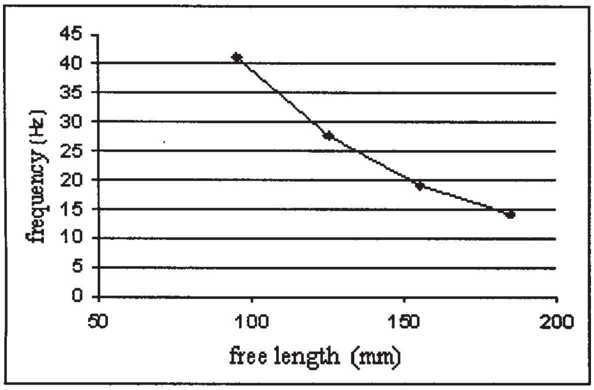

Fig. 8 Frequency of damped free vibrations, beam 1

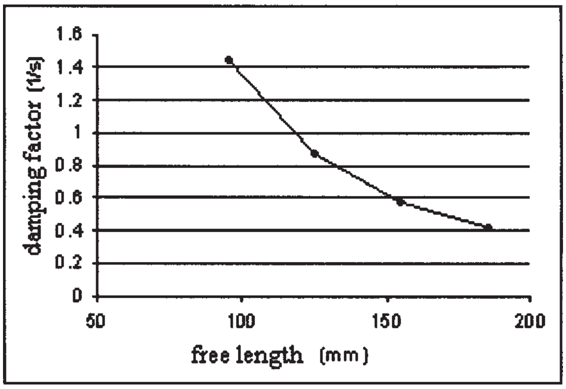

Fig. 9 Damping factor, beam 1

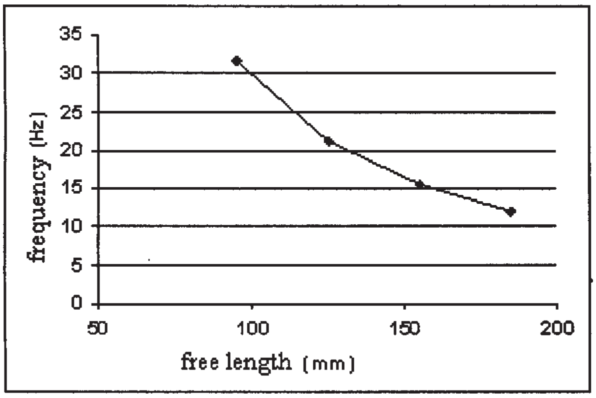

Fig. 10. Frequency of damped free vibrations, beam 2

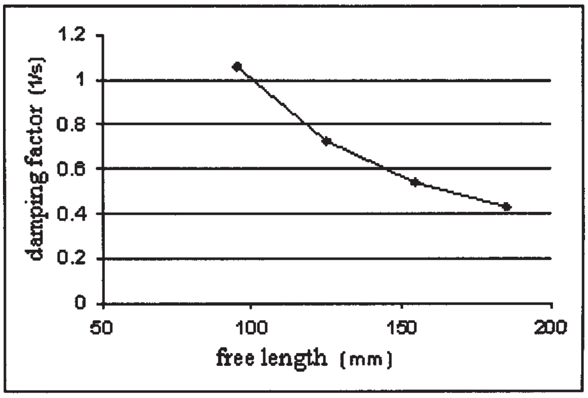

Fig. 11 Damping factor, beam 2

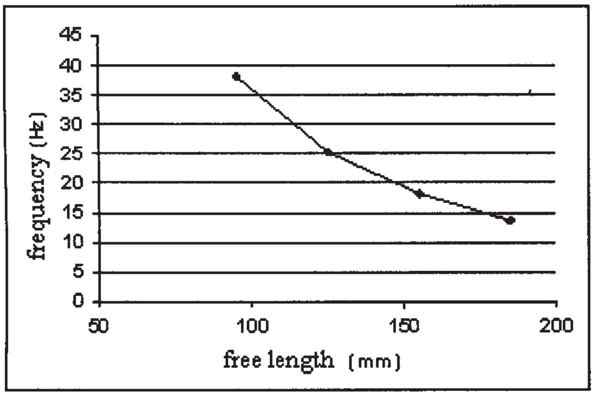

Fig. 12 Frequency of damped free vibrations, beam 3

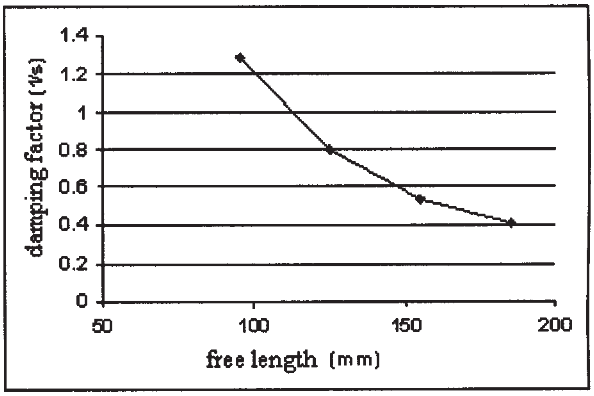

Fig. 13 Damping factor, beam 3

represents the initial amplitude of the damped free vibration. Since damping for the plates in damped free vibration is a combination of structural damping and damping due to friction with air, in all recordings for determining the damping factor μ, the areas in which the damping is between $X_1 = 0.2$ m/s^2 and $X_2 = 0.05$ m/s^2 were selected.

A spectral analysis, using the Finite Fourier Transform (FFT) techniques was undertaken and the frequency of damped free vibrations was obtained.

The record of the damped free vibrations for beam 1, with a free length of 185 mm is presented in figure 5, while in figure 6 the exponential curve used to determine the damping factor is plotted.

In figure 7 is shown a frequency spectrum, typical for beam 1 (one should underline that all records and spectra have similar characteristics).

Table 2

Material	c_1	λ_1	c_2	λ_2
1	1.1129	1.531	$175.2\cdot10^{-4}$	1.876
2	1.0140	1.463	$415.9\cdot10^{-4}$	1.378
3	1.0405	1.530	$211.4\cdot10^{-4}$	1.747

Forced vibrations with different initial amplitudes were induced for each plate, in order to observe the possible changes of the frequencies of vibration and damping. The frequency of vibration is practically independent of the amplitude of vibration, but damping depends on the instantaneous amplitude due to the friction with air. This is the reason for which the determination of the damping factor was made for vibrations having an amplitude between $X_1 = 0.2$ m/s^2 and $X_1 = 0.05$ m/s^2.

In figures 8 … 13 the graphs for damping factor and frequency of damped free vibrations as a function of the free length of the beams are plotted.

Conclusions

The obtained plots show that the variation of frequency with the length of the beam has the form:

$$\nu = \frac{c_1}{l^{\lambda_1}}. \qquad (13)$$

Theoretically, according to relationship (11), the exponent λ_1 should be 1.5.

Since the graphs for the damping factor have similar shapes with those for frequency, the following relationship for the damping factor is proposed by the authors:

$$\mu = \frac{c_2}{l^{\lambda_2}}. \qquad (14)$$

Since the damping factor is a material property, is should be determined experimentally. In table 2, the values of coefficients and exponents from relations (13) and (14) for the three studied materials are listed.

The values of the exponent λ_1 are very close to the theoretical one, showing that model with one degree of freedom can be successfully used to study the vibrations of a beam in a point. One can conclude that the natural frequency, for the first-mode of vibration, has the following features:

-increases proportionally with the thickness of the beam;

- increases with the Young's modulus;

-decreases when the specific mass increases.

Because the variations of the damping factor and of the frequency with the length of the beam are similar, one can draw similar conclusions for the variation of the damping factor.

Although relation (11) provides theoretical values of eigenfrequency very close to those obtained experimentally, the damping coefficient, as a characteristic of material can be determined only experimentally.

The decrease of the damping factor with the length of the beam can be explained by the fact that energy is dissipated in a bigger quantity of material.

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