

# Experimental Determinations of Stiffness and Damping Coefficient of the Sandwich Bars with Core of Polypropylene Honeycomb Reinforced with Steel Fabric

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*Starting from the dynamic response of a sandwich beam with damping, (which is in free vibration), it is established a method used to determine the damping factor. We experimentally calculated the stiffness and damping factor per unit mass for beams with middle layer made of polypropylene honeycomb respectively; the external layers were made of epoxy resin reinforced with steel fabric.*

*Keywords: sandwich beam, stiffness, damping factor, steel fabric, polypropylene honeycomb, free vibrations*

The composite plates and bars could be analyzed using a lot of theories that mostly differ by the inclusions or neglecting the effects of angular deformation and respectively, the rotational inertia.

Exact theories rely on a non-linear distribution of shear stresses along the thickness of the plate or bar. The inclusion of high order terms implies the inclusion of supplementary unknowns. Moreover, when fulfilling both the distribution of shear stresses in thickness is parabolic and if the limit conditions are accomplished on external surfaces, it is not necessary a correction factor. Based on this fact, it was developed a theory [1] (High – order Shear Deformation Theory – HSDT) where it is assumed that stresses and strains normal to the median plane are null. Another theory, in which there are also considered the stresses normal to the median plane, has also been developed in [2, 3] by removing a series of contradictions appearing in previous theories by accepting non linear factors of shear stresses in thickness; also, they did not neglect a part of the normal stresses obtained by the loading of the composite structure.

There have also been made some studies on the damped vibrations of Euler – Bernoulli and Timoshenko bar. Relevant to this works are the studies [4-11]. The material was assumed to be incompressible whereby the same viscoelastic operators could be both used for the flexural and shear deformations. This permitted the use of the normal modes and their orthogonality conditions to solve this viscoelastic forced vibration problem. It is shown that if the ratio between the length and thickness of a bar is higher than ten, the differences between Timoshenko and Euler-Bernoulli theories for the bending moment, shear force and the medium fiber deformation are smaller than five percent (5%). It is shown that, for the first vibration eigenmodes, the damping influence of the bar section rotational motion can be neglected. Similar equations and conclusions for composite materials bars are presented in [12-13]. In [9], it is analyzed a damped and axially loaded Timoshenko bar for random transverse load. Only a special

case of damping in the transverse and rotatory motion was considered which allowed, then, using the orthogonality conditions of the undamped modes to decouple the modal equations. In [10], it is obtained the „closed – form” solution, but for an incomplete differential equation of a simply – supported bar with external damping.

In [14], it is presented a general modal approach to solve the linear vibration problem of a uniform Timoshenko bar with external transverse and rotatory viscous damping and different viscoelastic damping in the flexural and shear deformations. With this approach, the bars with boundary conditions can be as conveniently analyzed as a bar with simple supports.

Recent applications have shown that honeycomb panels from polymer, reinforced with fiber, can be used for new construction or for restoration of existing structures. In [15], there are studied the vibrations of sandwich structures with honeycomb which have the core geometry of sinusoidal type. It was developed a higher order vibration model for studying the vibrations, made by energy methods.

In [16], there were studied the free vibrations of the curved sandwich beams, with flexible core, in different conditions of temperature. The external surfaces and the core of the beam were considered as being made of materials with mechanical properties dependent on temperature. It was shown that the frequency of free vibrations of the beams decreases when the temperature increases.

## Experimental part

We built the plates from composite materials with polypropylene honeycomb core (honeycomb which has the thickness  $g$  with the values 10 mm, 15 mm, and 20 mm). The exterior layers of the plates were made of epoxy resin reinforced with steel fabric.

For these plates, we collected six sets of samples with length equal with  $L = 400$  mm and width equal with  $l = 40$  mm and respectively  $l = 50$  mm.

These were noted as follows:

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- the set of samples 1: l = 40 mm, g = 10 mm;
- the set of samples 2: l = 50 mm, g = 10 mm;
- the set of samples 3: l = 40 mm, g = 15 mm;
- the set of samples 4: l = 50 mm, g = 15 mm;
- the set of samples 5: l = 40 mm, g = 20 mm;
- the set of samples 6: l = 50 mm, g = 20 mm;

We have also considered two variants of bar embedding, on various lengths, in this way (we will refer to the free parts of the plates – namely the parts where the accelerometers are located and where the measurements will be made):

- Variant I: the free length is 300 mm
- Variant II: the free length is 350 mm

We have chosen these lengths at each bar, to obtain the ratio between the bar length and thickness bigger than 15 and to be able to apply the Bernoulli theory which is valid in the case of thin bars. In this case, the free vibration of the bar is:

$$w(x,t) = \sum_{n=1} e^{-\mu t} \left( \frac{\mu}{2\pi\nu_n} \sin 2\pi\nu_n t + \cos 2\pi\nu_n t \right) \cdot V_n(x),$$

where:

- $\mu$  is the damping factor, equal with half of the damping factor per unit mass of the bar;
- $\nu_n$  are the eigenfrequencies;
- $V_n(x)$  are the eigenfunctions, which depend on the conditions of the bar ends.

The eigenfrequencies are calculated with the relations:

$$\nu_n = \frac{K_n}{2\pi l^2} \sqrt{\frac{EI}{m}}, \quad (1)$$

where

- l is the bar length;
- EI is the bar section stiffness;
- m is the mass per unit length of the bar;
- $K_n$  is determined from the bar ends conditions.

Experimental measurements have been made, recording the free vibrations in two measuring points. The measuring points (where the accelerometer was placed) are located at 10 mm (named P<sub>1</sub> point), respectively 100 mm (named P<sub>2</sub> point) distances, from the free end of the bar. Each point measurement was made four times. The data record for a bar of set 6, that has the free length of 300 mm and the measurement made in point P<sub>1</sub>, is presented in figure 1.

The processing of this data record and the calculus of the damping ratio, for a number of five cycles, are presented in figure 2. In this processing, we have determined half of the damping factor per unit mass of the bar.

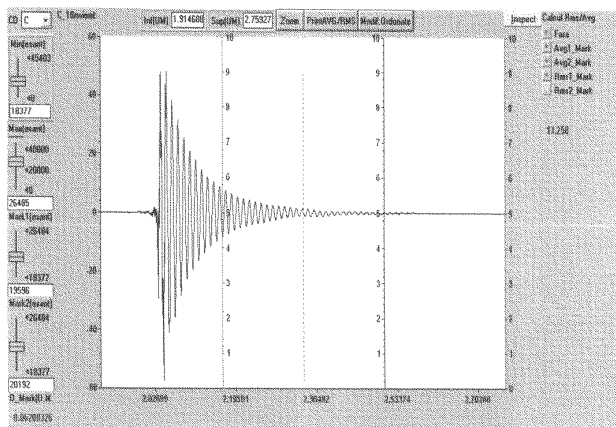


Fig.1

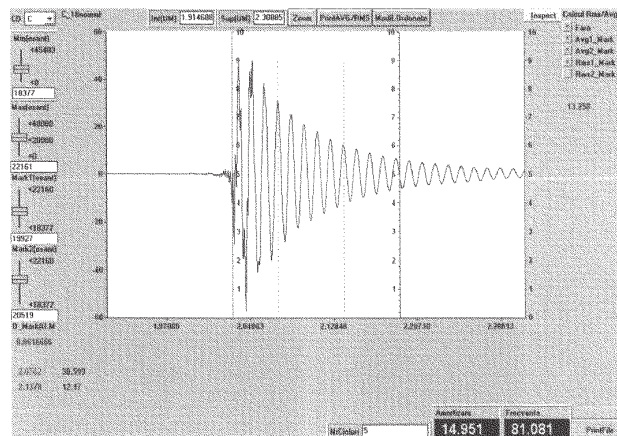


Fig. 2

Because we have not observed significant differences given by the point of measurement, we have made for each set of bars, the average values of the damping factor  $\mu$  (half of the damping factor per unit mass of the bar) for all the measurements. The experimentally determined results for mass per length bar unit, the damping factor, the frequency of the first eigenmode and the bar section stiffness obtained from relation (1) are presented in table 1.

Table 1

Set	m	Variant I			Variant II		
		$\mu$	$\nu$	EI	$\mu$	$\nu$	EI
1	0.185	8.40	45.84	10.06	6.87	34.55	10.58
2	0.236	8.47	45.93	12.88	6.43	32.79	12.16
3	0.201	13.36	62.22	20.13	9.47	47.95	21.41
4	0.251	12.42	61.54	24.59	8.73	46.01	25.36
5	0.210	16.43	79.73	34.54	11.87	59.46	35.58
6	0.272	14.95	80.95	46.11	11.34	59.84	46.68

In table 1, the measuring units are:

- for mass per length bar unit m-(kg / m);
- for the damping factor  $\mu$ -(Ns/ m)/ kg);
- for the frequency  $\nu$ -(s<sup>-1</sup>);
- for the bar section stiffness EI-(Nm<sup>2</sup>).

We tested the bar samples to bending for stiffness calculus.

Samples and test device were configured according to ASTM D790-02, Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastic and Electrical Insulating Materials. The testing speed has been calculated according to ASTM D790-02. The specifications of the test device are:

- the diameter of the support and indentor rollers - 25 mm;
  - the distance between the support rollers - 2 x 120 mm;
  - the width of support and indentor rollers - 50 mm
- In figure 3, there are shown the bending device and the way of the sample breakage.

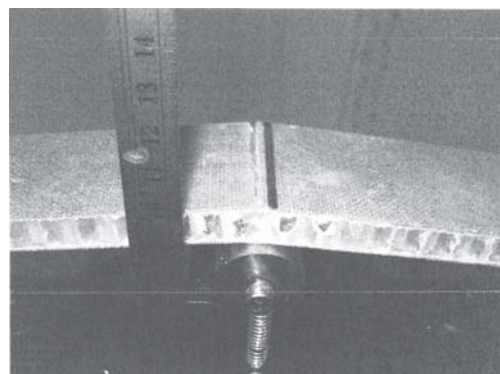


Fig. 3

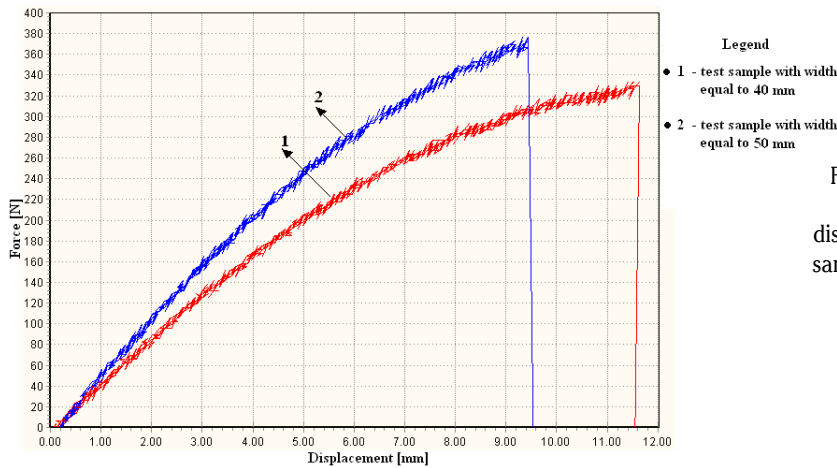


Fig. 4 The dependence between the force applied by the indentor roller and the displacement at the middle of the sample, for samples with polypropylene honeycomb core of 10 mm

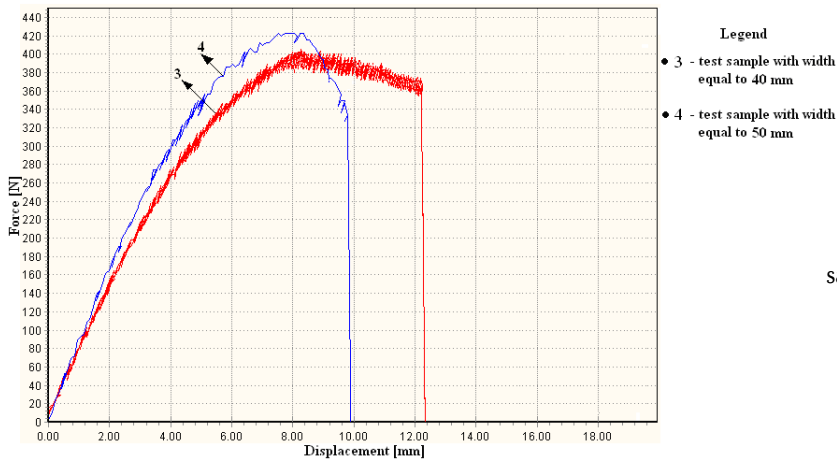


Fig. 5 The dependence between the force applied by the indentor roller and the displacement at the middle of the sample, for samples with polypropylene honeycomb core of 15 mm

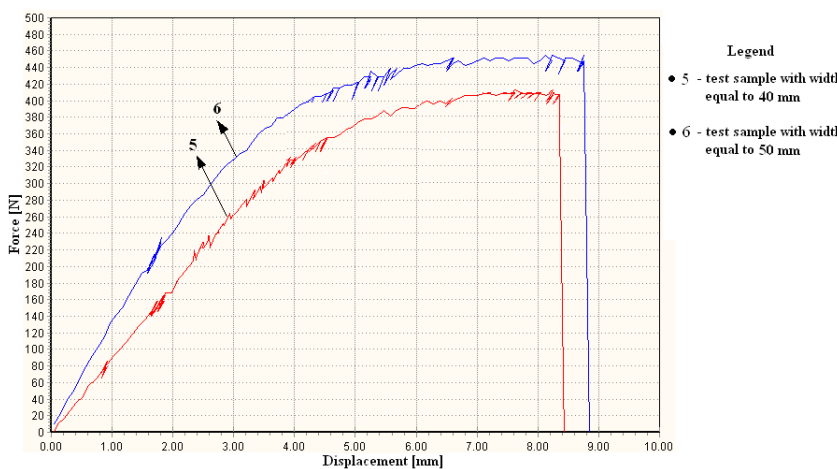


Fig. 6 The dependence between the force applied by the indentor roller and the displacement at the middle of the sample, for samples with polypropylene honeycomb core of 20 mm

The dependence between the force applied by the indentor roller and the displacement at the middle of the specimen is presented in figures 4-6.

From the diagrams 4-6, the bars section stiffness are determined by the force – displacement dependence interpolation in the linearity area which corresponds to some displacements up to 2 mm. We have obtained the next results:

- for set 1:  $EI = 11.24 \text{ Nm}^2$ ;
- for set 2:  $EI = 14.38 \text{ Nm}^2$ ;
- for set 3:  $EI = 20.30 \text{ Nm}^2$ ;
- for set 4:  $EI = 24.29 \text{ Nm}^2$ ;
- for set 5:  $EI = 32.91 \text{ Nm}^2$ ;
- for set 6:  $EI = 42.31 \text{ Nm}^2$

### Conclusions

The values analysis of damping coefficients indicates that these coefficients must be experimentally determined

for each type of material and sample, being difficult to deduce a quantitative correspondence with the parameters which influence the damping, directly or indirectly. The values of damping coefficients may depend on several factors such as: sample dimensions, specific mass or the quantity of material from sample, elastic and damping properties of component materials.

The sample width can influence the damping coefficient by the fact that it determines the surface in which the air friction is acting on the sample. The sample mass or specific linear mass has an influence on the damping coefficient by that, for the samples with higher mass and width, the deformation energy which is stored in the sample through the initial deformation, is dissipated in a larger quantity of material. An influence may occur due to the sample rigidity, explained by the fact that a force initially applied on the sample produces a less deformation if the rigidity is higher.

A good damping of vibrations is achieved in the case in which the composite materials of the external layers have the damping capacity and elastic properties which are superior. But the influence of these layers is dependent on the interaction with the middle layer and, for this reason, it is difficult to be analytically analyzed.

In addition to these general considerations, we can distinguish the following conclusions:

-both for damping coefficient per unit mass and as well for the damping coefficient per unit length of the bar, the highest values were obtained for bars with core thickness equal with 20 mm and the lowest values were obtained for bars with core thickness equal with 10 mm; therefore, the damping coefficient increases at once with the core thickness of bar;

-we have not observed the significant differences of the damping coefficient per unit mass of the bar, for the bars which have the width equal with 40 mm, and, respectively, for those which have the width equal with 50 mm;

-the damping factor values per bar mass unit decrease with the bar free length.

The comparison between the bars sections stiffness calculated from relation (1) and the experimental tests obtained at the three points bending shows that for the bars with polypropylene honeycomb core of 10 mm, the calculated stiffnesses are lower than the measured ones, and for the bars with polypropylene honeycomb core of 20 mm, the calculated rigidities are higher than the experimentally obtained ones.

The diagrams analysis that give the dependence between the force applied by the indentor roller and the displacement at the middle of the specimen, and the specimens bending tested, shows that the breakage is suddenly produced in the opposite part of the indentor roller that applies the loading force.

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