Study of Some Elastic Properties for Sandwich Bars with Symmetrical Distribution of Layers

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In this paper we determine a field of stress, which verifies the Cauchy equations of equilibrium, the conditions of continuity on the surfaces between layers, and boundary conditions, for a sandwich bar, with symmetrical distribution of layers, subject to traction. We have considered the relationships, previously obtained, for a composite bar built of three layers. Using a mediation formula for deformations and stress, we obtained a new formula for calculating the longitudinal elasticity modulus, in the case in which the constituent materials have different coefficients of transversal contraction. We realized the experimental measurements for test samples from polyesteric resin, reinforced with woven of fibers glass, carbon and glass-carbon.

Keywords: composite materials, characteristic curve, elasticity modulus

The composite plates and the bars may be analyzed using many theories that differ mostly by including or neglecting the effects of angular strain and rotational inertia respectively.

In composite materials containing fibers or particulate reinforcement the interface separating matrix from inclusion is widely believed to be a dominant influence affecting the overall stiffness and damage tolerance characteristics of the composite. Likewise damage accumulation in a composite often depends on the character of the mechanical response of the interface that, in the case of a weak interface, may precipitate such separation phenomena as brittle or ductile decohesion. In many fiber-reinforced composite systems the weak interfaces are somehow desirable since they generally raise the toughness, however, are often at the expense of composite stiffness. Accurate assessment of overall stiffness characteristics of a composite containing fiber weakly bonded to the matrix is therefore extremely important in the attempt to obtain improved composite performance.

Exact theories rely on a non-linear distribution of shear stress along the thickness of the plate or bar. The inclusion of high order terms implies the inclusion of supplementary unknowns. Moreover, when fulfilling both the parabolic distribution of shear stress in thickness and the limit conditions on external surfaces, a correction factor is not necessary anymore. Based on this fact, it was developed a theory in [1] (High – order Shear Deformation Theory – HSDT) where it is assumed that stress and strains normal to the median plane are null. Another theory in which the stress is normal to the median plane was considered too, has been developed in [2-3] and removes a series of contradictions appearing in previous theories by accepting non linear factors of shear stress in thickness; they didn't also neglect some of the normal stress obtained by the loading of the composite structure.

In [4] were obtained theoretical results and experimental determinations. Using a matrix method were determined the main elastic characteristics of composite materials and their variation depending on the volumetric proportion of reinforcement.

The studies of the composite materials dynamics reserved a special place for sandwich bars made from several overlapped layers with similar thickness. Most studies refer to three layer sandwich bars, the middle layer having visco-elastic behaviour and the inferior and superior layers having extra elastic and resilience proprieties. Other authors having similar studies on the behaviour of these materials suggested the following:

- there is a continuity of displacements and stress between layers;
- there is no deformation along the thickness of the bar;
- the transversal inertial forces are dominant, neglecting longitudinal inertia and rotational inertia of the bar section;
- the external layers have elastic behaviour and are subject to pure bending and the core has elastic or visco-elastic behaviour taking over shear stress;
- the core is not subject to normal stress.

Based on these hypotheses have been developed models considered to be the fundamentals of DTMM theory [5-7]. Considering this theory, it was adapted a variation approach, obtained equations for sandwich plates taking also into consideration different angular deformation for the layers and managing to estimate the stress between the layers [8].

Interlaminar stresses near free edges of composite bar are mainly responsible for delamination failures. Numerous studies have been undertaken to investigate interlaminar stress and failures of laminated composites. In [9] is studied the interlaminar tensile strength under static and fatigue loads including the temperature and moisture effects. In [10] is studied the effect of geometric nonlinearities on free-edge stress fields of bars. In [11] is investigated the response and failure for dropped-ply laminates tested in flat-end compression, and in [12] was shown that the times for delamination onset occurrences in composites can be predicted probabilistically.

Theoretically-obtained results

It is considered a composite bar with a rectangular section of width $2b$, made of 2p + 1 layers from different materials, with constant thickness along the length of the

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bar. Arrangement of layers is considered symmetrical to the median plane. We consider both a mass and elastic symmetry.

We report the bar to a reference system with axes oriented as follows:

- the axe *x* on the longitudinal direction of bar;
- the axe *y* on the width direction of the bar;
- the axe *z* on the thickness direction of bar.

The origin of the reference system is chosen in the median plane of the bar, which is also the median plane of layer, numbered with 1 (fig. 1).

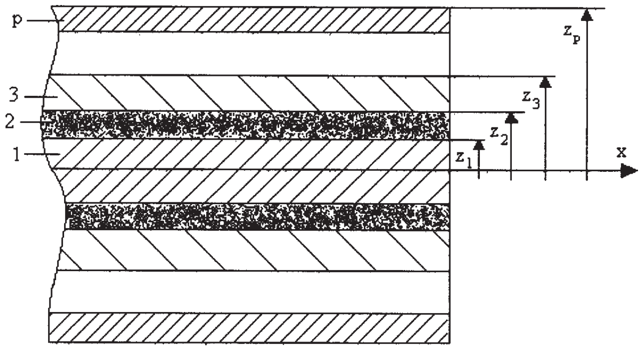

Fig. 1

The bar is subject to a load on the longitudinal direction. Because, in general, materials that make up the bar layers have different coefficients of transversal contraction, the state of stress shall be complex. If the length of the bar is high as compared to the transversal dimensions, we can accept that in the middle zone of the bar the stress that occurs does not depend on the coordinated *x*. In these conditions, the equilibrium Cauchy equations have the following form:

$$\frac{\partial \sigma_{xy}}{\partial y}+\frac{\partial \sigma_{xz}}{\partial z}=0,\quad \frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{yz}}{\partial z}=0,\quad \frac{\partial \sigma_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}=0. \tag{1}$$

Due to the symmetry must be satisfied conditions (relative to the axis y):

$$\sigma_{xy}(-y,z)=-\sigma_{xy}(y,z),\quad \sigma_{yz}(-y,z)=-\sigma_{yz}(y,z), \tag{2}$$

On the lateral surfaces of the bar we have:

$$\sigma_{xy}(b,z)=0,\quad \sigma_{yz}(b,z)=0. \tag{3}$$

Some of these conditions are fulfilled if $\sigma_{xz} = \sigma_{xz} = 0$. The remaining conditions are fulfilled if for the rest of the stress we consider the following representation in mathematical series (for each layer "i"):

$$\sigma_{yy}^{(i)}=\sum_{n=1}\frac{b}{n\pi}\left(12A_n^{(i)}z^2+2B_n^{(i)}\right)\left(\cos\frac{n\pi y}{b}-(-1)^n\right),$$
$$\sigma_{yz}^{(i)}=\sum_{n=1}\left(4A_n^{(i)}z^3+2B_n^{(i)}z\right)\sin\frac{n\pi y}{b}, \tag{4}$$
$$\sigma_{zz}^{(i)}=\sum_{n=1}\left(-\frac{n\pi}{b}\right)\left(A_n^{(i)}z^4+B_n^{(i)}z^2+C_n^{(i)}\right)\cos\frac{n\pi y}{b}.$$

Taking into account the forms of these components of stress tensor, in addition for the normal stress $\sigma_{xx}^{(i)}$ is considered the following form:

$$\sigma_{xx}^{(i)}=\sum_{n=1}\left[f_n^{(i)}(z)\cos\frac{n\pi y}{b}+g_n^{(i)}(z)\right]. \tag{5}$$

On separation surface between the layers "i" and "i + 1" (i=1,...,p - 1) must be satisfied the continuity conditions for stress, namely:

$$\sigma_{xz}^{(i)}(z_i)=\sigma_{xz}^{(i+1)}(z_i),\quad \sigma_{yz}^{(i)}(z_i)=\sigma_{yz}^{(i+1)}(z_i),\ i=\overline{1,p-1},\quad \sigma_{zz}^{(i)}(z_i)=\sigma_{zz}^{(i+1)}(z_i). \tag{6}$$

On the exterior surface of plate, the stress must be null, namely:

$$\sigma_{xz}^{(p)}(z_p)=0,\quad \sigma_{yz}^{(p)}(z_p)=0,\quad \sigma_{zz}^{(p)}(z_p)=0. \tag{7}$$

The continuity conditions for stress on the separation surface between the layers lead to the following relations:

$$2A_n^{(i)}z_i^2+B_n^{(i)}=2A_n^{(i+1)}z_i^2+B_n^{(i+1)},\ i=\overline{1,p-1},$$
$$A_n^{(i)}z_i^4+B_n^{(i)}z_i^2+C_n^{(i)}=A_n^{(i+1)}z_i^4+B_n^{(i+1)}z_i^2+C_n^{(i+1)}, \tag{8}$$

and on the exterior surface:

$$2A_n^{(p)}z_p^2+B_n^{(p)}=0,\quad A_n^{(p)}z_p^4+B_n^{(p)}z_p^2+C_n^{(p)}=0. \tag{9}$$

it results that:

$$B_n^{(p)}=-2A_n^{(p)}z_p^2,\quad C_n^{(p)}=A_n^{(p)}z_p^4,$$
$$B_n^{(i)}=-2A_n^{(i)}z_i^2-2\sum_{k=i+1}^{p}A_n^{(k)}\left(z_k^2-z_{k-1}^2\right),$$
$$C_n^{(i)}=A_n^{(i)}z_i^4+\sum_{k=i+1}^{p}A_n^{(k)}\left(z_k^4-z_{k-1}^4\right),\ i=\overline{1,p-1}. \tag{10}$$

The state of stress is determined with Hooke's law (it is considered that each of the layers behaves linearly elastic), more precisely:

$$
\begin{aligned}
\varepsilon_{xx}^{(i)} &= C_{11}^{(i)}\sum_{n=1}\left[f_n^{(i)}(z)\cos\frac{n\pi y}{b}+g_n^{(i)}(z)\right]+C_{12}^{(i)}\sum_{n=1}\frac{b}{n\pi}\left(12A_n^{(i)}z^2+B_n^{(i)}\right)\cdot\left(\cos\frac{n\pi y}{b}-(-1)^n\right)+\\
&+C_{13}^{(i)}\sum_{n=1}\left(-\frac{n\pi}{b}\right)\left(A_n^{(i)}z^4+B_n^{(i)}z^2+C_n^{(i)}\right)\cos\frac{n\pi y}{b},\\
\varepsilon_{yy}^{(i)} &= C_{12}^{(i)}\sum_{n=1}\left[f_n^{(i)}(z)\cos\frac{n\pi y}{b}+g_n^{(i)}(z)\right]+C_{22}^{(i)}\sum_{n=1}\frac{b}{n\pi}\left(A_n^{(i)}z^2+B_n^{(i)}\right)\cdot\left(\cos\frac{n\pi y}{b}-(-1)^n\right)+\\
&+C_{23}^{(i)}\sum_{n=1}\left(-\frac{n\pi}{b}\right)\left(A_n^{(i)}z^4+B_n^{(i)}z^2+C_n^{(i)}\right)\cos\frac{n\pi y}{b},\\
\varepsilon_{zz}^{(i)} &= C_{13}^{(i)}\sum_{n=1}\left[f_n^{(i)}(z)\cos\frac{n\pi y}{b}+g_n^{(i)}(z)\right]+C_{23}^{(i)}\sum_{n=1}\frac{b}{n\pi}\left(A_n^{(i)}z^2+B_n^{(i)}\right)\cdot\left(\cos\frac{n\pi y}{b}-(-1)^n\right)+\\
&+C_{33}^{(i)}\sum_{n=1}\left(-\frac{n\pi}{b}\right)\left(A_n^{(i)}z^4+B_n^{(i)}z^2+C_n^{(i)}\right)\cos\frac{n\pi y}{b},\\
\varepsilon_{xy}^{(i)} &= 0,\\
\varepsilon_{xz}^{(i)} &= 0, \qquad (11)\\
\varepsilon_{yz}^{(i)} &= C_{44}^{(i)}\sum_{n=1}\left(4A_n^{(i)}z^3+2B_n^{(i)}z\right)\sin\frac{n\pi y}{b}.
\end{aligned}
\tag{11}
$$

On the separation surface between the layers „i” and „i+1” (i =1,..., p-1), must be satisfied the continuity conditions for deformations, namely:

$$
\begin{aligned}
\varepsilon_{xx}^{(i)}(z_i) &= \varepsilon_{xx}^{(i+1)}(z_i),\\
\varepsilon_{yy}^{(i)}(z_i) &= \varepsilon_{yy}^{(i+1)}(z_i),\\
\varepsilon_{xy}^{(i)}(z_i) &= \varepsilon_{xy}^{(i+1)}(z_i).
\end{aligned}
\tag{12}
$$

From the continuity conditions for deformations on the separation surfaces between the layers it results that:

$$
\begin{aligned}
g_n^{(i)}(z_i) &= \frac{(-1)^n\frac{b}{n\pi}}{\Delta_i}\left[\left(C_{22}^{(i)}C_{11}^{(i+1)}-C_{12}^{(i)}C_{12}^{(i+1)}\right)\left(12A_n^{(i)}z_i^2+2B_n^{(i)}\right)-\right.\\
&\left.-\left(C_{22}^{(i+1)}C_{11}^{(i+1)}-C_{12}^{(i+1)}C_{12}^{(i+1)}\right)\left(12A_n^{(i+1)}z_i^2+2B_n^{(i+1)}\right)\right],
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
g_n^{(i+1)}(z_i) &= \frac{(-1)^n\frac{b}{n\pi}}{\Delta_i}\left[\left(C_{22}^{(i)}C_{11}^{(i)}-C_{12}^{(i)}C_{12}^{(i)}\right)\left(12A_n^{(i)}z_i^2+2B_n^{(i)}\right)-\right.\\
&\left.-\left(C_{22}^{(i+1)}C_{11}^{(i)}-C_{12}^{(i+1)}C_{12}^{(i)}\right)\left(12A_n^{(i+1)}z_i^2+2B_n^{(i+1)}\right)\right],
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
f_n^{(i)}(z_i) &= \frac{\frac{b}{n\pi}}{\Delta_i}\left[\left(C_{11}^{(i+1)}C_{22}^{(i+1)}-C_{12}^{(i+1)}C_{12}^{(i+1)}\right)\left(12A_n^{(i+1)}z_i^2+2B_n^{(i+1)}\right)-\right.\\
&\left.-\left(C_{11}^{(i+1)}C_{22}^{(i)}-C_{12}^{(i+1)}C_{12}^{(i)}\right)\left(12A_n^{(i)}z_i^2+2B_n^{(i)}\right)\right]-\\
&-\frac{\frac{n\pi}{b}}{\Delta_i}\left[\left(C_{11}^{(i+1)}C_{23}^{(i+1)}-C_{12}^{(i+1)}C_{13}^{(i+1)}\right)\left(A_n^{(i+1)}z_i^4+B_n^{(i+1)}z_i^2+C_n^{(i+1)}\right)-\right.\\
&\left.-\left(C_{11}^{(i+1)}C_{23}^{(i)}-C_{12}^{(i+1)}C_{13}^{(i)}\right)\left(A_n^{(i)}z_i^4+B_n^{(i)}z_i^2+C_n^{(i)}\right)\right],
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
f_n^{(i+1)}(z_i) &= \frac{\frac{b}{n\pi}}{\Delta_i}\left[\left(C_{11}^{(i)}C_{22}^{(i+1)}-C_{12}^{(i)}C_{12}^{(i+1)}\right)\left(12A_n^{(i+1)}z_i^2+2B_n^{(i+1)}\right)-\right.\\
&\left.-\left(C_{11}^{(i)}C_{22}^{(i)}-C_{12}^{(i)}C_{12}^{(i)}\right)\left(12A_n^{(i)}z_i^2+2B_n^{(i)}\right)\right]-\\
&-\frac{\frac{n\pi}{b}}{\Delta_i}\left[\left(C_{11}^{(i)}C_{23}^{(i+1)}-C_{12}^{(i)}C_{13}^{(i+1)}\right)\left(A_n^{(i+1)}z_i^4+B_n^{(i+1)}z_i^2+C_n^{(i+1)}\right)-\right.\\
&\left.-\left(C_{11}^{(i)}C_{23}^{(i)}-C_{12}^{(i)}C_{13}^{(i)}\right)\left(A_n^{(i)}z_i^4+B_n^{(i)}z_i^2+C_n^{(i)}\right)\right],\quad i=\overline{1,p-1},
\end{aligned}
\tag{16}
$$

where

$$
\Delta_i = C_{11}^{(i+1)}C_{12}^{(i)} - C_{11}^{(i)}C_{12}^{(i+1)},
\tag{17}
$$

in which $C_{11}^{(i)}, C_{22}^{(i)}, C_{12}^{(i)}, C_{13}^{(i)}, C_{23}^{(i)}$ are elastic constants for the material "i"

Due to the symmetry, the functions $g_n^{(i)}(z)$ and $f_n^{(i)}(z)$ must be uneven in *z* variable. Without lossing the generality, the functions $g_n^{(i)}(z)$ can be considered constants. In this case, for the layers 2,3,..., p-1, the values of the functions $g_n^{(i)}(z)$ must be verified the relations (13) and (14), so hence the constants $A_n^{(1)}, A_n^{(2)}, ..., A_n^{(p)}$ check p - 2 relations from type (13) and (14). Thus, only two of them are independent. In particular these can be $A_n^{(1)}$ and $A_n^{(2)}$.

In [13] are proposed the mediation formulae for the tension σ_{xx} and deformation ε_{xx}. With their help we shall determine the longitudinal elasticity modulus of the bar, given by formula:

$$
E_L = \frac{\sum_{n=1}^{p}\int_{z_{i-1}}^{z_i}\sigma_{xx}^{(i)}(z)dz}{\sum_{n=1}^{p}\int_{z_{i-1}}^{z_i}\varepsilon_{xx}^{(i)}(z)dz}.
\tag{18}
$$

The bar built from three layers

For a bar built from three layers (p=2) are obtained:

$$
\begin{aligned}
C_n^{(2)} &= A_n^{(2)}z_2^4,\\
B_n^{(2)} &= -2A_n^{(2)}z_2^2,\\
C_n^{(1)} &= A_n^{(1)}z_1^4 + A_n^{(2)}\left(z_2^4 - z_1^4\right),\\
B_n^{(1)} &= -2A_n^{(1)}z_1^2 - 2A_n^{(2)}\left(z_2^2 - z_1^2\right).
\end{aligned}
$$

If the layers which form the bar are homogenous and isotropic, then:

$$
\begin{aligned}
C_{11}^{(i)} &= C_{22}^{(i)} = C_{33}^{(i)} = \frac{1}{E_i},\\
C_{12}^{(i)} &= C_{13}^{(i)} = C_{23}^{(i)} = -\frac{\upsilon_i}{E_i},
\end{aligned}
\tag{19}
$$

where E_i is Young modulus of the material from layer „i”, and ν_i is the Poisson coefficient of the material from layer „i”.

If we neglect the higher order terms, the elasticity modulus (on longitudinal direction), calculated with relation (18) is:

$$
E_L = \frac{h_1(z_1)z_1 + h_2(z_1)(z_2 - z_1)}{\frac{1}{E_1}h_1(z_1)z_1 + \frac{1}{E_2}h_2(z_1)(z_2 - z_1)},
\tag{20}
$$

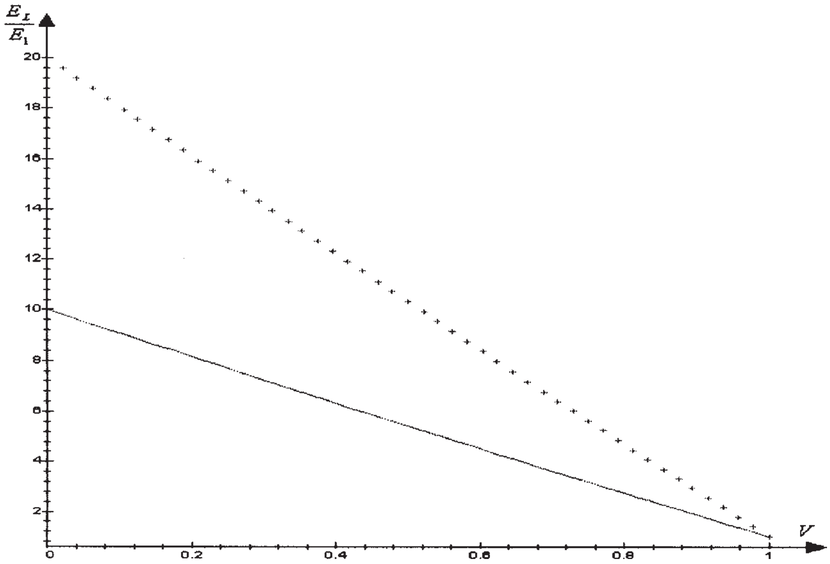

Fig.2

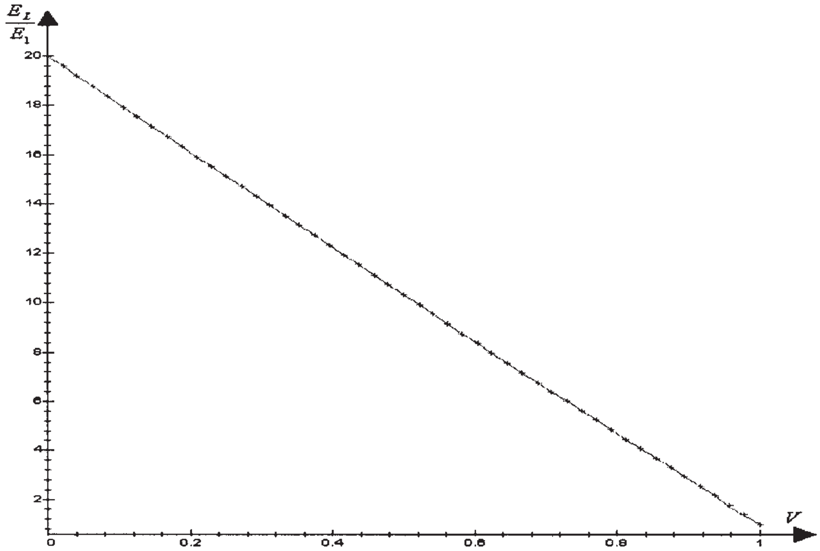

Fig.3

where

$$h_1(z_1) = \frac{E_1}{E_2}\left(1-\upsilon_2^2\right)\left(3z_1^2 - z_2^2\right) - \left(1-\upsilon_1\upsilon_2\right)\left(2\cdot x\cdot z_1^2 + z_1^2 - z_2^2\right),$$
$$h_2(z_1) = \left(1-\upsilon_1\upsilon_2\right)\left(3z_1^2 - z_2^2\right) - \frac{E_2}{E_1}\left(1-\upsilon_1^2\right)\left(2\cdot x\cdot z_1^2 + z_1^2 - z_2^2\right) \quad (21)$$

We note

$$x = \frac{A_n^{(1)}}{A_n^{(2)}}. \quad (22)$$

Parameter characterizes the way in which stress is distributed in section.

Using the notations:

$$\frac{E_1}{E_2} = e, \quad (23)$$

$z_1/z_2 = V$ (is volumetric proportion of the layer 1) we obtain for elasticity modulus E_L, the following expresion

$$\frac{E_L}{E_1} = \frac{e\cdot V\cdot p_1(e,V,x) + (1-V)\cdot p_2(e,V,x)}{e\cdot\left[V\cdot p_1(e,V,x) + (1-V)\cdot p_2(e,V,x)\right]}, \quad (24)$$

where

$$p_1(e,V,x) = e\cdot\left(3V^2-1\right)\left(1-\upsilon_2^2\right) - \left(1-\upsilon_1\upsilon_2\right)\left(2\cdot x\cdot V^2 + V^2 - 1\right),$$
$$p_2(e,V,x) = e\cdot\left(1-\upsilon_1\upsilon_2\right)\left(3V^2-1\right) - \left(1-\upsilon_1^2\right)\left(2\cdot x\cdot V^2 + V^2 - 1\right) \quad (25)$$

We can check immediately that if V = 1 (namely the bar is built just from material), it results that $E_l/E_1=1$, and hence $E_l=E_1$. If V = 0 (namely the bar is built just from material 2) it results that $E_l/E_1=1/e$, and hence $E_L=E_2$.

In the case in which the coefficients of transversal contraction are equal, we obtain:

$$\frac{E_L}{E_1} = V + \frac{1-V}{e}, \quad (26)$$

which is the classical formula from calculating the longitudinal elasticity modulus for unidirectional composite materials.

Therefore, the relationship for calculating the elasticity modulus given by (24) is consistent with both the basic physical considerations (if the bar is built just from material, then elasticity modulus given by (24), coincides with the elasticity modulus of the material from which is built the bar), as well as with the classical relationship for calculating the longitudinal elasticity modulus for composite materials.

In figure 2 is presented the variation of the $E_L=E_1$ ratio for ν_2 = 0.2; ν_1 = 0.3; x=0.05; e = 0.05 respectivelyex = 0.1 .

In figure 3 is presented the variation of the ratio $E_L=E_1$ for ν_2 = 0.2; ν_1 = 0.3; x=0.05; e = 0.05 respectively x = 1.

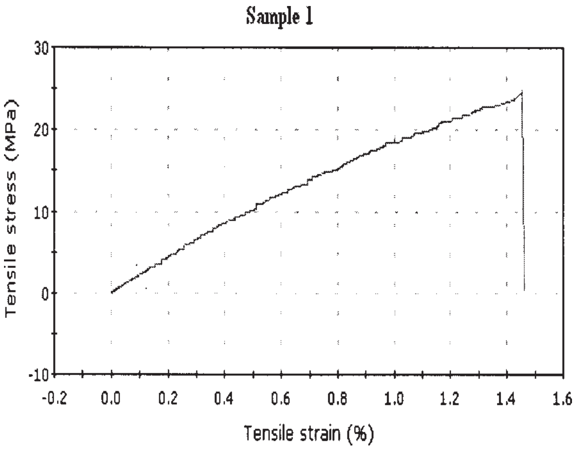

Fig.4

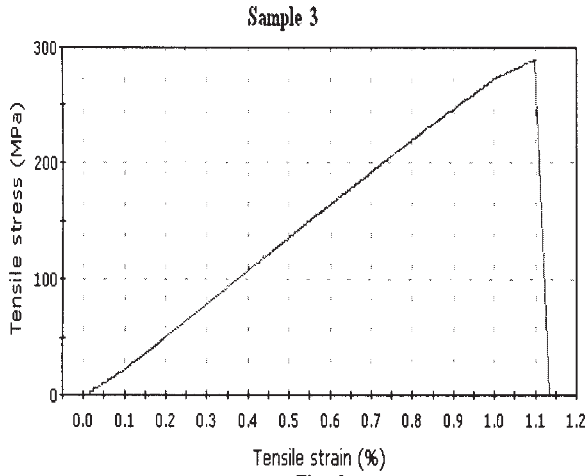

Fig. 6

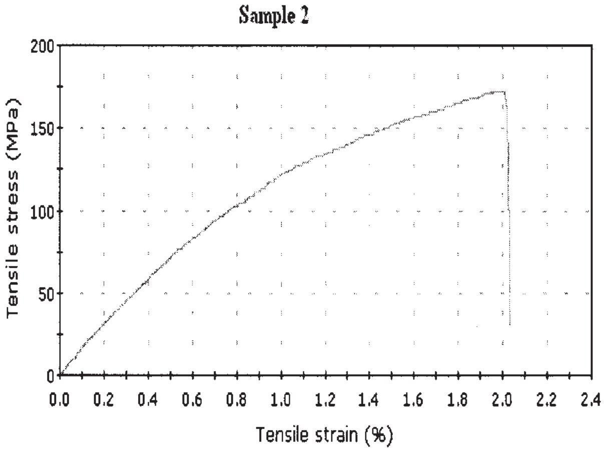

Fig.5

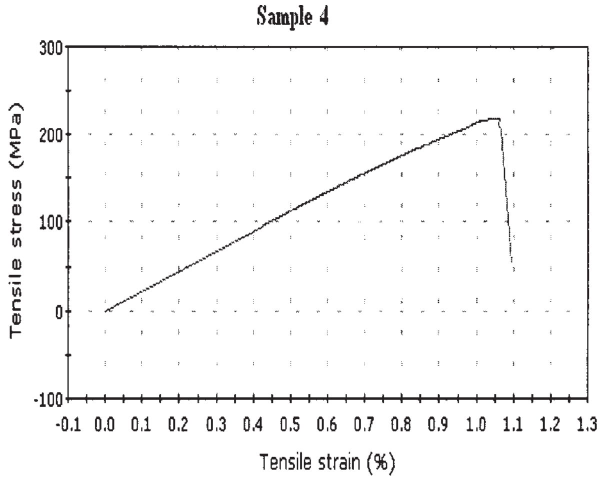

Fig.7

Table 1

Set of samples	Thickness (mm)	Width (mm)	Modulus (Automatic Young's) (MPa)	Modulus (Young's Tensile stress 0.02 % - 0.1 %) (MPa)
1	2.54000	9.17000	2181.75720	2938.32133
2	1.16000	9.62000	14999.61082	15483.51415
3	1.39000	8.51000	28629.73001	23014.99088
4	2.32000	9.45000	22816.29040	22688.00280

Table 2

Set of samples	Tensile stress at Yield (Offset 0.2 %) (MPa)	Tensile stress at Tensile Strength (MPa)	Tensile strain at Tensile Strength (%)
1	19.38734	24.38970	1.44771
2	119.78983	172.07108	1.99063
3	250.88417	284.59002	1.07520
4	197.46436	217.88483	1.04303

It is noted that the factor x has an insignificant influence, which can be verified even for x=1.

Experimental determinations

There were made four sets of samples as follows:

- the set of sample 1 built from polyester resin (with strengthener 3%);
- the set of sample 2 built from polyester resin reinforced with fiberglass fabric (two layers) with V=0.82;
- the set of sample 3 built from polyester resin reinforced with carbon fiber fabric (two layers) with V=0.88;
- the set of sample 4 built from polyester resin reinforced in the exterior layers with carbon fiber fabric (two layers) and in the median layer reinforced with fiberglass fabric (two layers) with V=0.5.

Volumetric proportions were determined by weighing the composite plates, and in parallel, by weighing the fibers which are into the composition of those plates, we obtained first of all the mass proportions of the components.

In figure 4 is presented the characteristic curve for a sample from the first set of samples, in figure 5 is given the characteristic curve for a sample from the second set of samples, in figure 6 is given the characteristic curve for a sample from the third set of samples, and in figure 7 is given the characteristic curve for a sample from fourth set of samples.

In tables 1 and 2 are presented experimental values obtained for the four sets of samples.

The elasticity modulus calculated with (24) is:

-for the material from polyester resin reinforced with fiber glass (set of sample 2)

$$E_L = 15124 \text{ MPa}$$

-for the material from polyester resin reinforced with carbon fiber (set of sample 3)

$$E_L = 29536 \text{ MPa}$$

-for the set of sample 4

.

$$E_L = 22330 \text{ MPa}$$

At the set of sample 4 we considered E_1 and E_2 as the theoretical elasticity modules, calculated for the sets of samples 2 and 3.

Conclusions

In the case of sandwich bars, with layers disposed symmetrically to the median plane, a request of traction leads to a symmetrical distribution of the stress.

In the case of bars built from three-layers, we can see that the variation of the elasticity modulus is almost linear with the volumetric proportion of median layer. The variation is even linear in the case in which transversal contraction coefficients for the median layer and outer layers are equal. If transversal contraction coefficients for the three layers are different then the variation of elasticity modulus of the composite, according to the volumetric proportion of median layer, is nonlinear. Graphics representations shows however, that we can apply the simplified relationship, deviations from linearity being very small.

Constitutive equation for composite materials in the form of sandwich bars with symmetrical distribution of the layers can be obtained by eliminating the parameter x. In general case the constitutive equation is nonlinear. If in the relations which contain the tension tensor components and deformation tensor components, are neglected the higher order terms, then we obtain a constitutive equation which is linear.

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Manuscript received: 3.11.2011