

# Rheological Model of the Neoprene Elements used for Base Isolation Against Seismic Actions

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*Construction of passive insulation elements in neoprene single microstructure mixture has demonstrated that the isolators can have the necessary elastic characteristics but low internal damping. The reduced scale models have in the structure viscoelastic materials basing on natural rubber symmetrically distributed in respect to the geometrical reference system chosen for the isolator. The materials consisting of natural or synthetic rubber are used as filling elements symmetrically located so that, globally, the isolator can be properly characterized both at compression and shear. For the present paper there were used two material types modelled as Voigt-Kelvin and Hooke-Maxwell elements.*

*Key words: rheological modelling, antivibrating isolator, composite neoprene*

Base isolation systems intended for seismic shocks and vibrations consist on rubber or neoprene viscoelastic elements constructed in various technical solutions. The most frequent passive isolation systems are constructed basing on neoprene elements in a sandwich construction having intermediary steel shims and the same neoprene mixture for each layer. [1, 2, 8]

In this case, the hysteretic dissipation factor of the isolator is equal to that for a single neoprene layer, all the layers being identical from the physico-mechanical and geometric point of view.

This study puts into evidence the possibility to realize and model the viscoelastic isolators based on microstructure composite mixture consisting on neoprene smoke black and chemical foam compound or solids with entrapped air (cork poudrette). Also, this approach points out the possibility to realize and model the macrostructural composites by assigning geometric spaces defined inside the isolator geometric space.

The rheological modelling for the neoprene isolator behaviour shows the increase of the internal energy dissipation by increasing of the hysteretic factor (damping structure coefficient) [5, 7].

## Structural composition of the neoprene isolator

The passive isolation elements for dynamic shocks and vibrations have to comply with the following conditions:

- provision of the bearing capacity by appropriate values of the mechanical resistance;
- provision of the rigidity necessary to attain static deflections under loading, conducting to the avoidance of the significant resonances with destroying character;

-optimization of the requirements concerning low rigidity with those for high internal dissipation.

In order to attain the above mentioned desiderates, a new approach regarding construction and modelling of the composite neoprene isolators is necessary. This approach will introduce an original concept namely, micro- and macrostructural composite antivibrating viscoelastic materials.

The viscoelastic materials consist on microstructural composite neoprene by using some appropriate grading of the powder materials – smoke black and air introduction in nanometric spaces, by cork poudrette or chemical foam compound. Thus, the composite neoprene at the microstructural level could be physically and technologically obtained and could be rheologically modelled as follows:

- microstructural composite neoprene having low damping expressed by the hysteretic factor  $\delta_1$  and low rigidity by the modulus of elasticity in shear  $G_1$  or rigidity factor  $k_1$ ;
- microstructural composite neoprene with high damping expressed by the hysteretic factor  $\delta_2$  and high rigidity by the modulus of elasticity in shear  $G_2$  or the rigidity factor  $k_2$ .

The macrostructural composite isolator consists on identical neoprene layers separated by steel shims, each layer consisting on macrostructural distinct units having various physico-mechanical characteristics. In this case, the neoprene elements are manufactured by controlled structure of the materials with  $\delta, k, j = 1, \dots, n$ , so that after curing the isolator could be obtained as in figure 1.

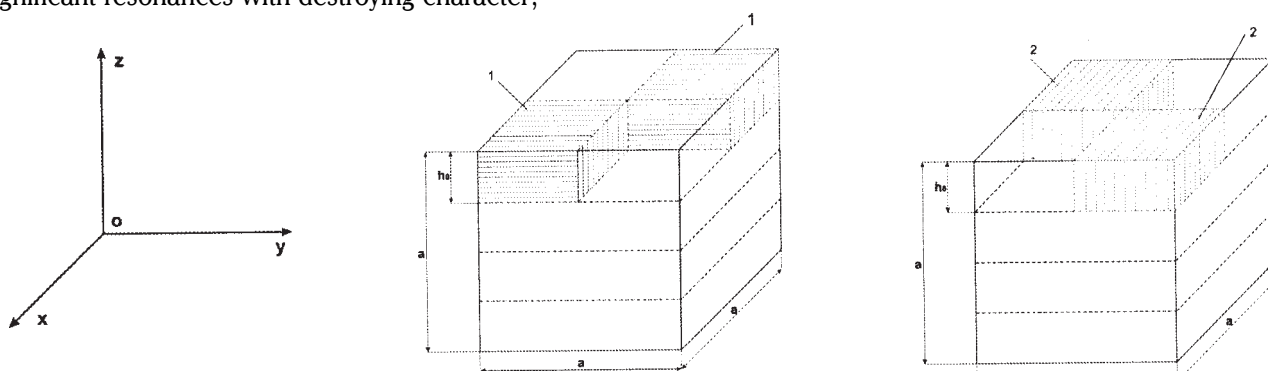


Fig 1. Isolator obtained after curing

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Structural parameters of the composite neoprene isolator  
Elastic and dissipation parameters for one macrostructural  
composite unit

Voigt-Kelvin model illustrated in figure 2 is characterized by the rigidity coefficient  $k$  and the structure dissipation coefficient (modulus) or hysteretic modulus  $\delta$ .

The complex dynamic rigidity is expressed under the form [4, 6]:

$$\tilde{K}(\omega) = k + j c \omega = k \left[ 1 + j \frac{c \omega}{k} \right]$$

or

$$\tilde{K}(\omega) = k(1 + j \delta) \tag{1}$$

where:

- $\delta$  is the hysteretic factor ( $\delta = c \omega / k$ );
- $c$  - viscous damping coefficient, Ns/m;
- $k$  - rigidity coefficient, N/m;
- $\omega$  - exciting pulsation (circular frequency), rad/s and
- $j = \sqrt{-1}$ .

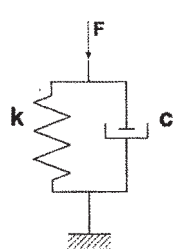


Fig. 2. Voigt-Kelvin model

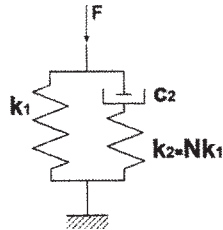


Fig. 3. Hooke-Maxwell model

Hooke-Maxwell model represented in figure 3 is characterized by the complex dynamic rigidity given by relation [2, 6]

$$\tilde{K}(\omega) = K_1(\omega) + j K_2(\omega) \tag{2}$$

where:

$$K_1(\omega) = \frac{k_1 k_2^2 + (k_1 + k_2) c^2 \omega^2}{k_2^2 + c^2 \omega^2} \tag{3}$$

is the dynamic elastic modulus, and

$$K_2(\omega) = \frac{k_2^2 c \omega}{k_2^2 + c^2 \omega^2} \tag{4}$$

is the internal energy loss modulus.

The internal dissipation coefficient or the hysteretic factor is defined under the form:

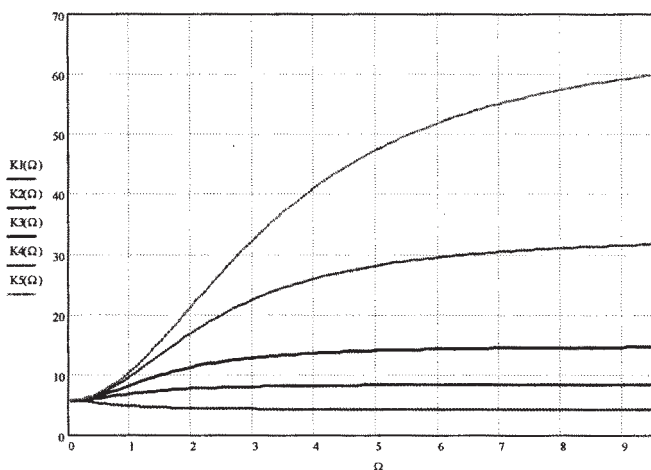


Fig. 4. - The graphic for  $K_1(\Omega)$

$$\Delta(\omega) = \frac{K_2(\omega)}{K_1(\omega)} = \frac{c k_2^2 \omega}{k_1 k_2^2 + (k_1 + k_2) c^2 \omega^2} \tag{5}$$

where we note with  $\alpha = \frac{k_1 + k_2}{k_1} = 1 + N$  the rigidity ratio;  
 $N = k_2 / k_1$ .

For the pulsation  $\omega_M = \frac{k_2}{c \sqrt{\alpha}}$  we have  $\Delta = \Delta_{max}$  expressed as:

$$\Delta = \Delta_{max} = \frac{\alpha - 1}{2 \sqrt{\alpha}} \tag{6}$$

Considering the pulsation ratio  $\omega / \omega_M = \Omega$  representing the relative exciting pulsation, the significant parametric relations for Hooke-Maxwell model are:

$$K_1(\Omega) = \frac{\alpha k_1 (1 + \Omega^2)}{\alpha + \Omega^2} \tag{7}$$

$$\Delta(\Omega) = 2 \Delta_{max} \frac{\Omega}{1 + \Omega^2} \tag{8}$$

parametrically illustrated in figures 4 and 5 where the parameters are  $\alpha$  and  $\Delta_{max}$ .

The complex dynamic rigidity for Hooke-Maxwell model can be written as:

$$\tilde{K}(\Omega) = K_1(\Omega) [1 + j \Delta(\Omega)] \tag{9}$$

Elastic and dissipation parameters for one macrostructural  
composite

Figure 6 presents the structure of a macrostructural composite viscoelastic layer, each unit being Voigt-Kelvin or Hooke-Maxwell modelled, all units being parallel connected.

In this case, the complex dynamic rigidity for more parallel connected elements is:

$$\tilde{K} = \sum_{i=1}^n K_i + j \sum_{i=1}^n K_i \Delta_i \tag{10}$$

where

- $i$  represents the order number for the individual element
- $n$  - the total number of the elements in one layer.

Relation (10) can be rewritten under the following form:

$$\tilde{K} = K_s (1 + j \Delta_s) \tag{11}$$

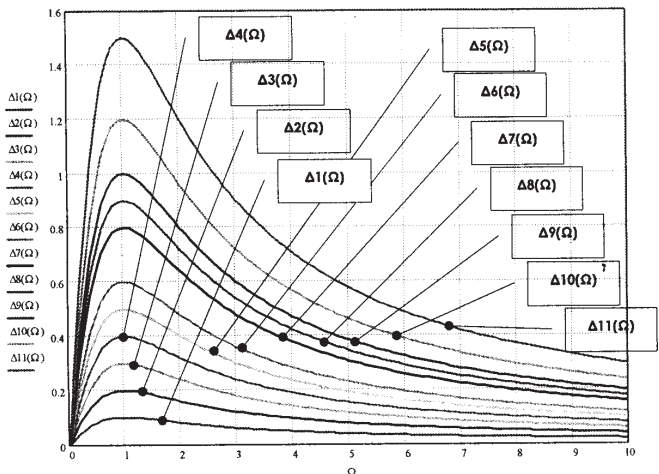


Fig. 5. The graphic for  $\Delta(\Omega)$

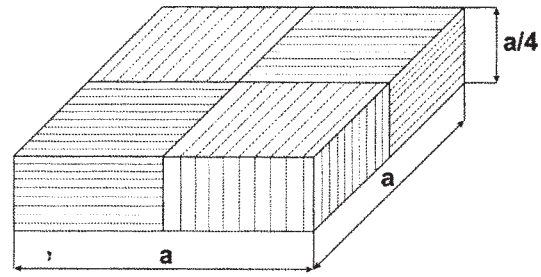
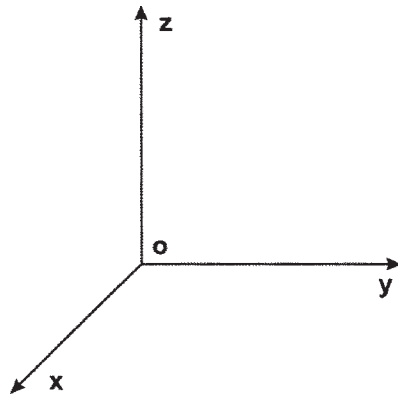


Fig. 6. The structure of a macrostructural composite viscoelastic layer

with  $K_s = \sum_{i=1}^n K_i$ , the total rigidity of one viscoelastic layer.

$$\Delta_s = \frac{\sum_i k_i \Delta_i}{\sum_i k_i} = \omega \frac{\sum_i c_i}{\sum_i k_i} \quad (12)$$

is the global hysteretic factor for one viscoelastic layer.

#### Elastic and dissipation parameters for one composite isolator

In case of an isolator consisting on  $s$  identical layers from the physical and geometrical point of view, separated by steel shims, serial connected, we have:

$$\begin{aligned} \tilde{K}_{isol} &= \frac{1}{s} \sum_{i=1}^n k_i + j \frac{1}{s} \sum_{i=1}^n k_i \Delta_i, \text{ or} \\ \tilde{K}_{isol} &= \frac{1}{s} \sum_{i=1}^n k_i \left[ 1 + j \frac{\sum_{i=1}^n k_i \Delta_i}{\sum_{i=1}^n k_i} \right] \end{aligned} \quad (13)$$

resulting in

$$K_{isol} = \frac{1}{s} \sum_{i=1}^n k_i \quad (14)$$

$$\Delta_{isol} = \Delta_s \quad (15)$$

Taking into account the critical damping  $\zeta$  and  $\Delta_1 = 2\zeta$ , we have:

$$\Delta_s = \Delta_{isol} = 2 \frac{\sum_i \zeta_i k_i}{\sum_i k_i} \quad (16)$$

#### *Performance analysis in case of composite neoprene isolator*

A rectangular isolator with  $a = b = h = 1$  m and four neoprene layers, consisting of two different antivibrating materials type SAB 31 having  $E_1 = 4.3$  MN/m<sup>2</sup>,  $G_1 = 0.8$  MN/m<sup>2</sup>, 55°ShA,  $\delta_1 = 0.17$ ,  $\zeta_1 = 0.085$  and SAB 4a with  $E_2 = 7.0$  MN/m<sup>2</sup>,  $G_2 = 1.16$  MN/m<sup>2</sup>, 65°ShA,  $\delta_2 = 0.25$ ,  $\zeta_2 = 0.125$  is considered.

For each viscoelastic unit with the dimensions  $a/2$ ,  $a/2$ ,  $h_0 = a/4$  and the shape coefficient  $\Phi = 0.5$ , we have  $K_1^x = 5.8$  MN/m for compression,  $K_1^y = 0.8$  MN/m for shearing,  $K_2^z = 9.4$  MN/m for compression and  $K_2^x = 1.16$  MN/m for shearing.

Table 1 presents the synthesis for three rheological models using Voigt-Kelvin, where the composite neoprene becomes of about four times more elastic while maintaining the hysteretic dissipation and thus, providing the dynamic isolation optimization.

Table 2 illustrates the results of Hooke-Maxwell model, where the composite neoprene becomes of about four times more elastic and the global dissipation increases with 33%, this being a favourable solution for the resonance vicinity.

Table 3 presents the results of Voigt-Kelvin and Hooke-Maxwell complex model having the elasticity lower with 6% and the hysteretic dissipation lower with 24% than Hooke-Maxwell simple model. This solution can be intended for dynamic isolation at postresonance.

#### **Results and discussions**

The tests have been carried out under dynamic regime using a special stand designed and patented in Romania. Thus, the tests have been performed under actual conditions related to the dynamic loading so that the response of the neoprene element corresponds to the significant operating parameters.

In this context, the neoprene elements have been constructed using the rubber mix SAB31 manufactured by Romanian specialized factories in Jilava and Pitești. The insulating elements have been tested as follows:

- insulating element consisting on four neoprene layers SAB31 having the hardness 55°ShA and intermediary steel shims, denoted IE4-S;
- insulating element consisting on four macrostructural neoprene layers basing on rubber SAB31, having the hardness 65°ShA and intermediary steel shims, denoted IV-K4-C1;
- insulating element consisting on four macrostructural neoprene layers basing on rubber SAB31M, having the hardness 51°ShA and 65°ShA, respectively with intermediary steel shims, denoted IH-M4-C2.

The rigidity and damping parameters have been experimentally determined by means of the resonance method under steady dynamic regime as well as by eigen vibration evaluation in case of system free motion after the perturbing force stops its action. The relations between the specific parameters are the following [3, 5, 7, 9]:

$$\zeta = \frac{1}{2\pi} \delta \quad (17)$$

$$\beta = \Delta = 2\zeta \quad (18)$$

$$\Psi = 4\pi\zeta = 2\pi\Delta \quad (19)$$

$$K_x^{din} = 4\pi^2 f_0^2 m \quad (20)$$

where:

$\delta$  is the logarithmic decrement;

$\zeta$  - the critical damping fraction;

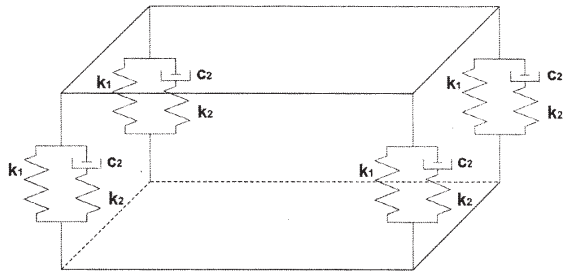
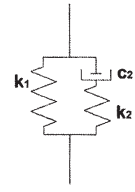
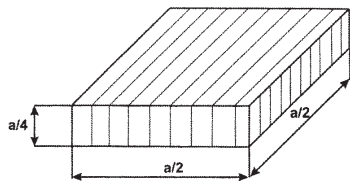
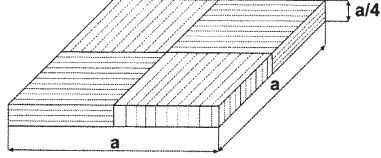
$\beta$  - the structural (hysteretic) loss coefficient;  
 $\Delta$  - the loss angle;  
 $\Psi$  - the energy dissipation coefficient;  
 $K_x^{din}$  - the dynamic rigidity factor;  
 $f_0^{din}$  - the eigen (natural) frequency;  
 $m$  - mass of mechanical system exciting the stand mobile part.

The experimental results are presented in table 4.

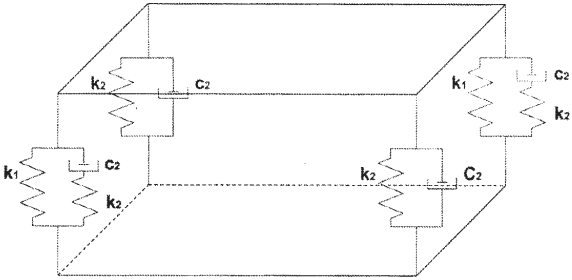
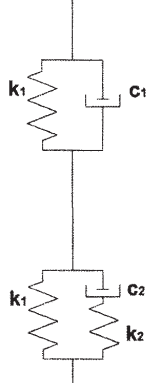
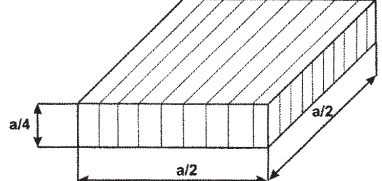
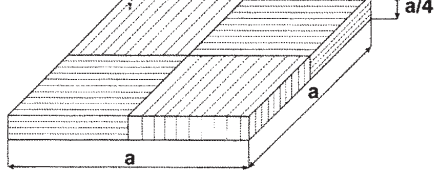
**Table 1**  
VOIGT - KELVIN MODEL

<p><b>a)</b> for one viscoelastic layer</p>	<p><b>b)</b> for a viscoelastic unit with <math>a/2, a/2, a/4</math></p>
<p>Viscoelastic unit</p>	<p>shape coefficient <math>\Phi = 0,5</math>          rubber SAB 31 for <math>(K_1, c_1)</math> and SAB 4a for <math>(K_2, c_2)</math>  <math>\tilde{K}_1 = K_1(1 + j\delta_1)</math>; <math>\tilde{K}_2 = K_2(1 + j\delta_2)</math>          complex rigidity: coefficients for compression <math>\tilde{K}_1^z, \tilde{K}_2^z</math> and shearing <math>\tilde{K}_1^x, \tilde{K}_2^x</math></p>
<p>Viscoelastic layer</p>	<p>rigidity coefficient</p> <ul style="list-style-type: none"> <li>• compression <math>K_s^z = 2(K_1^z + K_2^z) = 304 \text{ MN/m}</math></li> <li>• shearing <math>K_x = 2(K_1^x + K_2^x) = 3,92 \text{ MN/m}</math></li> </ul> <p>hysteretic dissipation factor  <math>\Delta_s^{V-K} = 2 \frac{K_1 \zeta_1 + K_2 \zeta_2}{K_1 + K_2} \Rightarrow \Delta_{s_z}^{V-K} = \Delta_{s_x}^{V-K} = 0,218</math></p>
<p>Sandwich isolator with 4 viscoelastic layers</p>	<p>rigidity coefficient</p> <ul style="list-style-type: none"> <li>• compression <math>K_z = \frac{30,4}{4} = 7,6 \text{ MN/m}</math></li> <li>• shearing <math>K_x = \frac{3,92}{4} = 0,98 \text{ MN/m}</math></li> </ul> <p>hysteretic dissipation factor <math>\Delta_z^{V-K} = \Delta_s^{V-K} = 0,218</math></p>

**Table 2**  
HOOKE-MAXWELL MODEL

 <p>a) for one viscoelastic layer</p>	 <p>b) for a viscoelastic unit</p>
<p>Viscoelastic unit</p>	 <p>rubber type SAB 22 and SAB 4a rigidity coefficient</p> $K_1^{H-M} = \frac{\alpha K_1 (1 + \Omega^2)}{\alpha + \Omega^2}; \alpha = 1 + N; N = \frac{K_2}{K_1}; \Omega = \frac{\omega}{\omega_M};$ $\omega_M = \frac{K_2}{c_2 \cdot \sqrt{1 + N}} = \frac{k_2}{c_2 \cdot \sqrt{\alpha}}$ <p>ratio <math>\alpha = 1 + 2\Delta_{max}^2 + 2\Delta_{max} \sqrt{1 + \Delta_{max}^2}</math></p> <ul style="list-style-type: none"> <li>for <math>\Delta_{max} = 0,5</math> results in <math>\alpha = 2,61</math></li> <li>rigidity coefficient <math>K_2 = (\alpha - 1)K_1; K_2 = 1,61 \cdot 5,8 \cong 9,4 \frac{MN}{m}</math></li> </ul> <p>viscous damping coefficient <math>c_2</math></p> $c_2 = \frac{K_2}{\omega_M \sqrt{\alpha}} = \frac{K_2}{2\pi f \sqrt{\alpha}}; c_2 = \frac{9,4}{2\pi \cdot 5 \cdot \sqrt{2,61}} = 0,185 \frac{MN \cdot s}{m}$ <p>compression for <math>\Omega = 1</math> <math>K_{1z}^{H-M} = \frac{2,61 \cdot 5,8 \cdot (1 + 1)}{2,61 + 1} = 8,38 \text{ MN/m}</math></p> <p>shearing for <math>\Omega = 1</math> <math>K_{1x}^{H-M} = \frac{2,61 \cdot 0,8 \cdot (1 + 1)}{2,61 + 1} = 1,15 \text{ MN/m}</math></p> <p>hysteretic dissipation factor</p> $\Delta_x^{H-M} = \Delta_z^{H-M} = 2 \cdot \frac{(\alpha - 1) \cdot \zeta_2}{1 + 4 \cdot \alpha \cdot \zeta_2^2};$ $\Delta_x^{H-M} = \Delta_z^{H-M} = 2 \cdot \frac{1,61 \cdot 0,375}{1 + 4 \cdot 2,61 \cdot 0,375^2} \cong 0,5$ <p>microstructural composite rubber</p> <ul style="list-style-type: none"> <li>hardness 65 °Sh</li> <li>longitudinally modulus <math>E = 7,0 \text{ MN/m}^2</math></li> <li>shear modulus <math>G = 1,16 \text{ MN/m}^2</math></li> </ul> <p><math>\delta_2 = 0,75; \zeta = 0,375</math> modified structure</p>
<p>Viscoelastic layer</p>	 <p>rigidity coefficient</p> <ul style="list-style-type: none"> <li>compression <math>K_{1z}^{H-M} = 4K_{1z} = 4 \cdot 8,38 = 33,52 \text{ MN/m}</math></li> <li>shearing <math>K_{1x}^{H-M} = 4K_{1x} = 4 \cdot 1,15 = 4,60 \text{ MN/m}</math></li> </ul> <p>hysteretic dissipation factor <math>\Delta_{1z}^{H-M} = 0,5</math></p>
<p>Sandwich isolator</p>	<p>rigidity coefficient</p> <ul style="list-style-type: none"> <li>compression <math>K_z^{H-M} = \frac{33,52}{4} = 8,38 \text{ MN/m}</math></li> <li>shearing <math>K_x^{H-M} = \frac{4,6}{4} = 1,15 \text{ MN/m}</math></li> </ul> <p>hysteretic dissipation factor <math>\Delta_{1z}^{H-M} = 0,50; \zeta_{1z} = 0,25</math></p>

**Table 3**  
VOIGT-KELVIN AND HOOKE-MAXWELL MODEL

	<p>V - K model    H - M model</p> 
<p>Viscoelastic unit</p>	 $\tilde{K}^{V-K} = K_1^{V-K} (1 + j\Delta_1^{V-K}), \quad \tilde{K}^{H-M} = K_1^{H-M} (1 + j\Delta_1^{H-M})$ <p>where:</p> $K_{1z}^{V-K} = 5,8 \text{ MN/m}; \Delta_1^{V-K} = 0,218; K_{1z}^{H-M} = 8,38 \text{ MN/m}; \Delta_1^{H-M} = 0,50$ $K_{1x}^{V-K} = 0,8 \text{ MN/m}; K_{1x}^{H-M} = 1,15 \text{ MN/m}$ $\Delta_s^x = \frac{(2K_1^{V-K})\Delta_1^{V-K} + (2K_1^{H-M})\Delta_1^{H-M}}{(2K_1^{V-K} + 2K_1^{H-M})}; \Delta_s^z = \frac{2 \cdot 5,8 \cdot 0,218 + 2 \cdot 8,38 \cdot 0,5}{(2 \cdot 5,8 + 2 \cdot 8,38)} = 0,38;$ $\Delta_s^x = \frac{2 \cdot 0,8 \cdot 0,218 + 2 \cdot 1,15 \cdot 0,5}{2 \cdot 0,8 + 2 \cdot 1,15} = 0,38$
<p>Viscoelastic layer</p>	 $\tilde{K}_s^c = 2\tilde{K}^{V-K} + 2\tilde{K}^{H-M}; \tilde{K}_s^c = 2[(K_1^{V-K} + K_1^{H-M}) + j(\Delta_1^{V-K} + \Delta_1^{H-M})]$ $K_s^z = 2(5,8 + 8,38) = 28,36 \text{ MN/m}; K_s^x = 2(0,8 + 1,15) = 3,90 \text{ MN/m}$
<p>4 layers sandwich isolator</p>	<p>rigidity coefficient</p> <ul style="list-style-type: none"> <li>• compression <math>K_z^c = \frac{1}{4} K_s^c = \frac{1}{4} \cdot 28,36 = 7,9 \text{ MN/m}</math></li> </ul>
	<ul style="list-style-type: none"> <li>• shearing <math>K_x^c = \frac{1}{4} K_s^x = \frac{1}{4} \cdot 3,90 = 0,975 \text{ MN/m}</math></li> </ul> <p>internal dissipation of the energy</p> <ul style="list-style-type: none"> <li>• hysteretic dissipation factor <math>\Delta_z^c = \Delta_x^c = 0,380</math></li> <li>• critical damping fraction <math>\zeta = 0,190</math></li> </ul>

**Table 4**

Sample	$K_x^{dyn}$ [MN/m]	$f_0$ [Hz]	$\delta$	$\zeta$	$\Delta = \beta$	$\Psi$
IE4-S	0,85	1,03	0,37	0,06	0,12	0,75
IV-K4-C1	1,05	1,15	0,69	0,11	0,22	1,38
IH-M4-C2	1,35	1,30	1,50	0,24	0,48	3,01

## Conclusions

By rheological modelling of the micro and macrostructural dissipative isolators in composite neoprene, high performances as global elasticity as well as internal dissipation can be attained. Thus, we conclude as follows:

- the isolator rigidity in case of compression for the first two models remains almost the same meaning that  $K_z^{V-K} = 7,6$  MN/m,  $K_x^{V-K} = 0,98$  MN/m for the Voigt - Kelvin model and  $K_z^{H-M} = 8,38$  MN/m,  $K_x^{H-M} = 1,15$  MN/m for the Hooke-Maxwell model;

- the internal dissipation expressed by the hysteretic factor is much higher for the neoprene composite Hooke-Maxwell modelled as compared with Voigt-Kelvin model,

meaning  $\Delta_{isolator}^{H-M} = 0,50$  and  $\Delta_{isolator}^{V-K} = 0,218$ ;

- the rigidity coefficients in case of composite neoprene isolator with Voigt-Kelvin and Hooke-Maxwell elements have lower values than the Hooke Maxwell model and the hysteretic damping  $\Delta_{isolator}^c = 0,380$ .

Basing on the case studies optimum and efficient solution for base isolation systems consisting of micro and macrostructural composites could be found.

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