Exact Relationships in Phenomenological Analysis of Rheodynamic Relaxation / Retardation Processes in Linear Viscoelasticity of Polymer Systems

II.2. Secondary Characteristic Quantities in the Case of Stress - Controlled Processes

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The secondary rheodynamic quantities in case of stress (σ)-controlled retardation processes, including the the storage, loss and absolute moduli, as well as the corresponding loss factor, are considered from the viewpoint of frequency dependence, exact relationships providing the- σ - characteristic frequencies corresponding to maximum and inflection points being given for the (1,1) - standard linear viscoelastic model. The results are illustrated in natural, semi-logarithmic, logarithmic and double-logarithmic coordinate representations.

Keywords: Exact relationships, rheodynamic retardation, linear viscoelasticity, polymer systems

Appropriate assessment of rheological behaviour of polymer materials in the framework of linear viscoelasticity strongly recommends the use of dynamic characterization [1-9]. Accordingly, the two distinct lines involving the strain(ε)- and stress(σ)-controlled processes, respectively, providing in a straight manner primary and secondary rheodynamic quantities, are to be considered together with the modulus-compliance interrelationships. Finally, the comprehensive (primary-secondary) description being accomplished in the *ε-controlled* case [1,3], that of σ-*controlled* one [2] is considered for the secondary counterpart for different typical coordinate representations used in the circumstances of standard linear viscoelastic model.

General definitions

In the general case of *solid linear viscoelastic behaviour*, the rheological equation for the (m,n)- *rheological state* can be expressed in terms of P, Q differential operators as

$$Q_{(n)}\varepsilon = P_{(m)}\sigma \qquad \text{(II.1)}$$

where

$$Q_{(n)} = q_0 + q_1 D_t + q_2 D_t^{(2)} + \ldots + q_n D_t^{(n)} \qquad \text{(II.2.1)}$$

$$P_{(m)} = p_0 + p_1 D_t + p_2 D_t^{(2)} + \ldots + p_m D_t^{(m)} \qquad \text{(II.2.2)}$$

and ε, σ denote the *natural rheological variables*, $q_0, q_1, q_2, \ldots q_n$; $p_0, p_1, p_2, \ldots, p_m$ are the *nominal rheological parameters*, while $D_t^{(r)} = d^r / dt^r$ represents the r-th order time derivative operator, and $m \le n$.

As previously established [2], in the case of a *sinusoidal stress-controlled process* of given amplitude, σ_o, and circular frequency, ω,

$$\tilde{\sigma} = \sigma_0 \exp(i\,\omega\, t) \qquad \text{(II.3)}$$

the corresponding *steady-state strain* is of the form

$$\tilde{\varepsilon} = \varepsilon_0(\omega)\exp\{i[\omega t - \delta_{(m,n)}(\omega)]\} \qquad \text{(II.4)}$$

and, by definition

$$\tilde{\varepsilon} / \tilde{\sigma} = [\varepsilon_o(\omega)/\sigma_o]_{(m,n)}\{\cos[\delta_{(m,n)}(\omega)] - i\sin[\delta_{(m,n)}(\omega)]\} \qquad \text{(II.5)}$$

is the *complex compliance*

$$J^*_{(m,n)}(\omega) = J'_{(m,n)}(\omega) - iJ''_{(m,n)}(\omega) \qquad \text{(II.6)}$$

whereas in the case of a *sinusoidal strain-controlled process* of given amplitude, ε_0, and circular frequency, ω, [1],

$$\tilde{\varepsilon} = \varepsilon_0 \exp(i\,\omega\, t) \qquad \text{(II.7)}$$

the corresponding *steady-state stress* is of the form

$$\tilde{\sigma} = [\sigma_0(\omega)]_{(m,n)} \exp\{i[\omega t + \delta_{(m,n)}(\omega)]\} \qquad \text{(II.8)}$$

and, by definition

$$\tilde{\sigma} / \tilde{\varepsilon} = [\sigma_o(\omega)/\varepsilon_o]_{(m,n)}\{\cos[\delta_{(m,n)}(\omega)] + i\sin[\delta_{(m,n)}(\omega)]\} \qquad \text{(II.9)}$$

denotes the *complex modulus*

$$M^*_{(m,n)}(\omega) = M'_{(m,n)}(\omega) + iM''_{(m,n)}(\omega) \qquad \text{(II.10)}$$

In the case of σ- *controlled* dynamic process when the complex compliance, $J^*_{(m,n)}$, results in a direct manner, along with that of the ε-*controlled* one, when the corresponding complex modulus, $M^*_{(m,n)}$, is provided, the key relationship occurs

$$J^*_{(m,n)}(\omega) M^*_{J(m,n)}(\omega) = 1 \qquad \text{(II.11)}$$

In case of σ-controlled processes, the complex compliance, $J^*_{(m,n)}$, (ω) is given by *direct primary rheodynamic quantities* - the (m,n)-storage compliance,

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$J'_{(m,n)}$, (ω), and the (m,n)-loss compliance, $J''_{(m,n)}$, (ω), or by using the *derived primary rheodynamic quantities* - the (m,n)-absolute compliance, $|J^*_{(m,n)}$, (ω)|, and the (m,n)-loss factor, $\beta_{(J,m,n)}$, (ω)- whereas the corresponding complex modulus, $M^*_{J(m,n)}$, (ω), results in terms of *secondary rheodynamic quantities* - the storage modulus $M'_{J(m,n)}$, (ω), and the loss modulus, $M''_{J(m,n)}$, (ω), defined as

$$M^*_{J(m,n)}(\omega) = M'_{J(m,n)}(\omega) + iM''_{J(m,n)}(\omega) \qquad \text{(II.12)}$$

which can be calculated by using the relationships

$$M'_{J(m,n)}(\omega) = J'_{M(m,n)}(\omega)/|J^*_{M(m,n)}|^2 \qquad \text{(II.13)}$$

$$M''_{J(m,n)}(\omega) = J''_{M(m,n)}(\omega)/|J^*_{M(m,n)}|^2 \qquad \text{(II.14)}$$

as well as those for the absolute modulus, $|M^*_{J(m,n)}$, (ω)|, given as

$$|M^*_{J(m,n)}| = [M'^2_{J(m,n)}(\omega) + M''^2_{J(m,n)}(\omega)]^{1/2} \qquad \text{(II.15)}$$

and the corresponding loss factor, $\beta_{MJ(m,n)}$ (ω) assigned by

$$\beta_{M_{J(m,n)}}(\omega) = \tan\delta_{M_{J(m,n)}}(\omega) = M''_{J(m,n)}(\omega)/M'_{J(m,n)}(\omega) =$$
$$= J''_{M(m,n)}(\omega)/J'_{M(m,n)}(\omega) = \tan\delta_{J_{M(m,n)}}(\omega) = \beta_{J_{M(m,n)}}(\omega) \qquad \text{(II.16)}$$

σ - (1, 1) rheological state frequency effects

For the model of *standard linear viscoelastic solid* which states the situation when m=n=1, i. e., the mono-relaxant/ mono-retardant case, the arising rheological equation of the (1,1)-*rheological state* is, [2],

$$q_0\varepsilon + q_1 D_t\varepsilon = p_0\sigma + p_1 D_t\sigma \qquad \text{(II.17)}$$

The use of definition relationships,(II.10, II.11), /2/, and (II.13-II.16) provides the set of *secondary* σ-*controlled rheodynamic quantities* - $M'_{J(1,1)}$, $M''_{J(1,1)}$, $|M^*_{J(1,1)}(\omega)|$ and $\beta_{M_{J(m,n)}}(\omega)$ on the basis of *primary* ones.

In terms of *nominal* rheological parameters results

$$M'_{J(1,1)}(\omega) = (p_0q_0 + p_1q_1\omega^2)/(p_0^2 + p_1^2\omega^2) \qquad \text{(II.18.1)}$$

$$M''_{J(1,1)}(\omega) = (p_0q_1 - p_1q_0)\omega/(p_0^2 + p_1^2\omega^2) \qquad \text{(II.18.1)}$$

$$|M^*_{J(1,1)}(\omega)| = [(q_0^2 + q_1^2\omega^2)/(p_0^2 + p_1^2\omega^2)]^{1/2} \qquad \text{(II.18.1)}$$

$$\beta_{M_{J(1,1)}}(\omega) = (p_0q_1 - p_1q_0)\omega/(p_0q_0 + p_1q_1\omega^2) \qquad \text{(II.18.1)}$$

and if the *characteristic* rheological parameters

$$C_0 = p_0/q_0, C_1 = p_1/q_0, D_1 = q_1/q_0 \qquad \text{(II.19)}$$

are used, one gets

$$M'_{J(1,1)}(\omega) = (C_0 + C_1D_1\omega^2)/(C_0^2 + C_1^2\omega^2) \qquad \text{(II.20.1)}$$

$$M''_{J(1,1)}(\omega) = (C_0D_1 - C_1)\omega/(C_0^2 + C_1^2\omega^2) \qquad \text{(II.20.2)}$$

$$|M^*_{J(1,1)}(\omega)| = [(1 + D_1^2\omega^2)/(C_0^2 + C_1^2\omega^2)]^{1/2} \qquad \text{(II.20.3)}$$

$$\beta_{M_{J(1,1)}}(\omega) = (C_0D_1 - C_1)\omega/(C_0 + C_1D_1\omega^2) \qquad \text{(II.20.4)}$$

Given the *limit* values for storage modulus are

$$\lim_{\omega\to 0} M'_{J(1,1)}(\omega) = M'_0 = 1/C_0 = 1/J'_0 \qquad \text{(II.22.1)}$$

$$\lim_{\omega\to\infty} M'_{J(1,1)}(\omega) = M'_\infty = D_1/C_1 = 1/J'_\infty \qquad \text{(II.22.2)}$$

and $D_1(=\tau_\sigma)$ is defined as a time dimension quantity, traditionally termed as the "*retardation time*", even if it is visible that the $D_1 = 1/\omega_\tau$ definition, where ω_τ is a *characteristic frequency* seems to be more suitable, functional relationships arise.

Accordingly,

$$M'_{J(1,1)}(\omega) = (J'_0 + J'_\infty\tau_\sigma^2\omega^2)/(J'^2_0 + J'^2_\infty\tau_\sigma^2\omega^2) \qquad \text{(II.22.1)}$$

$$M''_{J(1,1)}(\omega) = (J'_0 - J'_\infty)\tau_\sigma\omega/(J'^2_0 + J'^2_\infty\tau_\sigma^2\omega^2) \qquad \text{(II.22.2)}$$

$$|M^*_{J(1,1)}(\omega)| = [(1 + \tau_\sigma^2\omega^2)/(J'^2_0 + J'^2_\infty\tau_\sigma^2\omega^2)]^{1/2} \qquad \text{(II.22.3)}$$

$$\beta_{M_{J(1,1)}}(\omega) = (J'_0 - J'_\infty)\tau_\sigma\omega/(J'_0 + J'_\infty\tau_\sigma^2\omega^2) \qquad \text{(II.22.4)}$$

Furthermore, given the basic frequency significance of $1/\tau_\sigma$,

$$M'_{J(1,1)}(\omega) = (J'_0 + J'_\infty\omega^2/\omega_\sigma^2)/(J'^2_0 + J'^2_\infty\omega^2/\omega_\sigma^2) \qquad \text{(II.23.1)}$$

$$M''_{J(1,1)}(\omega) = [(J'_0 - J'_\infty)\omega/\omega_\sigma]/(J'^2_0 + J'^2_\infty\omega^2/\omega_\sigma^2) \qquad \text{(II.23.2)}$$

$$|M^*_{J(1,1)}(\omega)| = [(1 + \omega^2/\omega_\sigma^2)(J'^2_0 + J'^2_\infty\omega^2/\omega_\sigma^2)]^{1/2} \qquad \text{(II.23.3)}$$

$$\beta_{M_{J(1,1)}}(\omega) = [(J'_0 - J'_\infty)\omega/\omega_\sigma]/(J'_0 + J'_\infty\omega^2/\omega_\sigma^2) \qquad \text{(II.23.4)}$$

ω_σ being the frequency at which the loss compliance presents a *maximum value*, and can be defined as the *natural frequency* of the σ *controlled dynamic retardation process*. Finally,

$$M'_{J(1,1)}(\varpi_\sigma) = (J'_0 + J'_\infty\varpi_\sigma^2)/(J'^2_0 + J'^2_\infty\varpi_\sigma^2) \qquad \text{(II.24.1)}$$

$$M''_{J(1,1)}(\varpi_\sigma) = [(J'_0 - J'_\infty)\varpi]/(J'^2_0 + J'^2_\infty\varpi^2) \qquad \text{(II.24.2)}$$

$$|M^*_{J(1,1)}(\varpi_\sigma)| = [(1 + \varpi_\sigma^2)(J'^2_0 + J'^2_\infty\varpi_\sigma^2)]^{1/2} \qquad \text{(II.24.3)}$$

$$\beta_{M_{J(1,1)}}(\varpi_\sigma) = [(J'_0 - J'_\infty)\varpi_\sigma]/(J'_0 + J'_\infty\varpi_\sigma^2) \qquad \text{(II.24.4)}$$

where $\varpi_\sigma = \omega/\omega_\sigma$ represents the corresponding *relative (normalized) frequency*.

σ - (1, 1) characteristic frequencies

In order to identify the main consequences of the obtained set of secondary rheo-dynamic quantities in case of σ-*controlled* processes, the corresponding first- ,D_ω, and second-order, $D_\omega^{(2)}$, frequency derivative are calculated, the solution of resulting equations providing the *necessary conditions* of maximum and inflection, respectively.

For the storage modulus, it follows

$$D_\omega M'_{J(1,1)}(\omega) = 2C_0C_1(C_0D_1 - C_1)\omega/(C_0^2 + C_1^2\omega^2)^2 \qquad \text{(II.25.1)}$$

$$D_\omega^{(2)} M'_{J(1,1)}(\omega) = 2C_0C_1(C_0D_1 - C_1)(C_0^2 - 3C_1^2\omega^2)/(C_0^2 + C_1^2\omega^2)^3 \qquad \text{(II.25.2)}$$

the inflection frequency yielding as

$$\omega_{i(M_{J(1,1)})} = sol[D_\omega^{(2)} M'_{J(1,1)}(\omega) = 0] = (\sqrt{3}/3)(C_0/C_1)_1 =$$
$$= (\sqrt{3}/3)(J'_0/J'_\infty)(1/\tau_\sigma) = (\sqrt{3}/3)(J'_0/J'_\infty)\omega_\sigma \qquad \text{(II.26.1)}$$

In case of the loss modulus one obtains

$$D_\omega M''_{J(1,1)}(\omega) = (C_0 D_1 - C_1)(C_0^2 - C_1^2\omega^2)/(C_0^2 + C_1^2\omega^2)^2 \quad \text{(II.27.1)}$$

$$D_\omega^{(2)} M''_{J(1,1)}(\omega) = 2C_1^2 (C_0 D_1 - C_1)(-3C_0^2 + C_1^2\omega^2)\omega/(C_0^2 + C_1^2\omega^2)^3 \quad \text{(II.27.2)}$$

the corresponding maximum frequency being

$$\omega_{m\{M''_{J(1,1)}\}} = sol[D_\omega M''_{J(1,1)}(\omega) = 0] = C_0/C_1 = (J'_0/J'_\infty)(1/\tau_\sigma) = = (J'_0/J'_\infty)\omega_\sigma \quad \text{(II.28.1)}$$

whereas the inflection one is

$$\omega_{i\{M''_{J(1,1)}\}} = sol[D_\omega^{(2)} M''_{J(1,1)}(\omega) = 0] = \sqrt{3}(C_0/C_1) = = \sqrt{3}(J'_0/J'_\infty)(1/\tau_\sigma) = \sqrt{3}(J'_0/J'_\infty)\omega_\sigma \quad \text{(II.29.1)}$$

For the absolute modulus one gets

$$D_\omega |M^*_{J(1,1)}(\omega)| = (C_0^2 D_1^2 - C_1^2)\omega/[(C_0^2 + C_1^2\omega^2)^{3/2}(1 + D_1^2\omega^2)^{1/2}] \quad \text{(II.30.1)}$$

$$D_\omega^{(2)} |M^*_{J(1,1)}(\omega)| = (C_0^2 D_1^2 - C_1^2)(C_0^2 - 2C_1^2\omega^2 - 3C_1^2 D_1^2\omega^4)/ /[(C_0^2 + C_1^2\omega^2)^{5/2}(1 + D_1^2\omega^2)^{3/2}] \quad \text{(II.30.2)}$$

resulting for inflection

$$\omega_{i\{M_J\}} = \{sol^*[D_\omega^{(2)} M_J = 0] \rightarrow sol^*[3C_1^2 D_1^2\omega^4 + 2C_1^2\omega^2 - C_0^2 = 0]\} = = (1/3C_1 D_1)\sqrt{3C_1(\sqrt{3C_0^2 D_1^2 + C_1^2} - C_1)} = = (\sqrt{3}/3)(1/D_1)\sqrt{\sqrt{1 + 3C_0^2/(C_1^2/D_1^2)} - 1} = = (\sqrt{3}/3)\omega_\tau \sqrt{\sqrt{1 + 3J'^2_0/J'^2_\infty} - 1} \quad \text{(II.31.1)}$$

In the case of the corresponding loss factor

$$D_\omega \beta_{M_{J(1,1)}}(\omega) = (C_0 D_1 - C_1)(C_0 - C_1 D_1\omega^2)/(C_0 + C_1 D_1\omega^2)^2 \quad \text{(II.32.1)}$$

$$D_\omega^{(2)} \beta_{M_{J(1,1)}}(\omega) = 2C_1 D_1 (C_0 D_1 - C_1)(-3C_0 + C_1 D_1\omega^2)\omega/(C_0 + C_1 D_1\omega^2)^3 \quad \text{(II.32.2)}$$

the maximum condition gives

$$\omega_{m\{\beta_{M_{J(1,1)}}\}} = sol[D_\omega \beta_{M_{J(1,1)}}(\omega) = 0] = \sqrt{C_0/C_1 D_1} = \sqrt{J'_0/J'_\infty}(1/\tau_\sigma) = = \sqrt{J'_0/J'_\infty}\,\omega_\sigma \quad \text{(II.33.1)}$$

while for the inflection one results

$$\omega_{m\{\beta_{M_{J(1,1)}}\}} = sol[D_\omega \beta_{M_{J(1,1)}}(\omega) = 0] = \sqrt{C_0/C_1 D_1} = \sqrt{J'_0/J'_\infty}(1/\tau_\sigma) = = \sqrt{J'_0/J'_\infty}\,\omega_\sigma \quad \text{(II.33.2)}$$

As result of the explicit sequence of values

$$\sqrt{J'_0/J'_\infty} < \sqrt{3J'_0/J'_\infty} < (\sqrt{3}/3)(J'_0/J'_\infty) < J'_0/J'_\infty < \sqrt{3}(J'_0/J'_\infty) \quad \text{(II.34.1)}$$

and in virtue of the fact that for polymeric materials $J'_0 > 10J'_\infty$, the resulting physical consequence

$$\omega_{i\{M_J\}} < \omega_{m\{\beta_{M_J}\}} \langle \omega_{i\{\beta_{M_J}\}} \langle \omega_{i\{M'_J\}} \langle \omega_{m\{M''_J\}} \langle \omega_{i\{M''_J\}} \quad \text{(II.34.2)}$$

expresses the existence, in the case of σ - *controlled processes*, for different secondary rheo dynamic quantities of a *full* set of σ - *characteristic frequencies*. The relationships for the frequencies corresponding to the complete set of primary and secondary viscoelastic quantities are given in table 1.

Table 1

CHARACTERISTIC FREQUENCIES CORRESPONDING TO PRIMARY AND SECONDARY RHEODYNAMIC QUANTITIES FOR σ - CONTROLLED PROCESSES

Viscoelastic Quantity	Frequency	Value
J'	$\omega_{i\{J'\}}$	$(\sqrt{3}/3)\omega_\sigma$
J''	$\omega_{m\{J''\}}$	ω_σ
J''	$\omega_{i\{J''\}}$	$\sqrt{3}\omega_\sigma$
J	$\omega_{i\{J\}}$	$(\sqrt{3}/3)(J'_0/J'_\infty)\omega_\sigma\sqrt{\sqrt{1+3J'^2_\infty/J'^2_0}-1}$
BJ	$\omega_{m\{\beta_J\}}$	$\sqrt{J'_0/J'_\infty}\,\omega_\sigma$
BJ	$\omega_{i\{\beta_J\}}$	$\sqrt{3}\sqrt{J'_0/J'_\infty}\,\omega_\sigma$
M'_J	$\omega_{i\{M'_J\}}$	$(\sqrt{3}/3)(J'_0/J'_\infty)\omega_\sigma$,
M''_J	$\omega_{m\{M''_J\}}$	$(J'_0/J'_\infty)\omega_\sigma$
M''_J	$\omega_{i\{M''_J\}}$	$\sqrt{3}(J'_0/J'_\infty)\omega_\sigma$
M_J	$\omega_{i\{M_J\}}$	$(\sqrt{3}/3)\omega_\sigma\sqrt{\sqrt{1+3J'^2_0/J'^2_\infty}-1}$
B_{M_J}	$\omega_{m\{\beta_{M_J}\}}$	$\sqrt{J'_0/J'_\infty}\,\omega_\sigma$
B_{M_J}	$\omega_{i\{\beta_{M_J}\}}$	$\sqrt{3}\sqrt{J'_0/J'_\infty}\,\omega_\sigma$

Taking into account the results obtained for ε- and σ-*controlled* primary rheodynamic quantities, [1, 2], the two basic relationships,

$$\omega_\varepsilon = (J'_0/J'_\infty)\omega_\sigma \quad \text{(II.35)}$$

$$\omega_\sigma \omega_\varepsilon = \omega_{\sigma\varepsilon}^2 \quad \text{(II.36)}$$

among many others, are pointed out, where ω_ε represents the *natural frequency of the* ε- *controlled rheodynamic process*, to which corresponds the maximum of loss modulus, and $\omega_{\sigma\varepsilon}$ is the σε - characteristic *mixed frequency* corresponding to the maximum value of loss factor, each quantity providing a distinct criterion for appropriate assessing of the material behaviour potential.

Simulation of secondary (1, 1) - rheodynamic effects

In order to point out the peculiarities of frequency dependence in the case of different secondary σ-*controlled* rheodynamic processes, physically realistic values of J'_0, J'_∞, ω_σ are considered, e.g.,

$$J'_0 = 1[1/MPa], J'_\infty = 10^{-3}[1/MPa], \omega_\sigma = 1, 2, 5[rad/s]$$

The obtained results are given in figure II.1 for the *storage modulus* (M1J), in figure II.2 – for the *loss modulus* (M2J), as well as in figure II.3 for the *absolute modulus* (MJ), and in figure II.4 – for the corresponding *loss factor* (BMJ), taking into account different standard coordinate representations - natural(a), semi-logarithmic(b), logarithmic(c) and double-logarithmic(d).

The calculated values of characteristic frequencies for given σ-*controlled* dynamic processes are given in table 2.

The dissimilar features of frequency dependence of different σ - *controlled secondary rheodynamic quantities* in case of (1, 1) - rheological behavior, draw attention, on the hand, for the *storage modulus* (fig. II.1), on the increase with increasing frequency, from the *low value*, M'_0, at zero

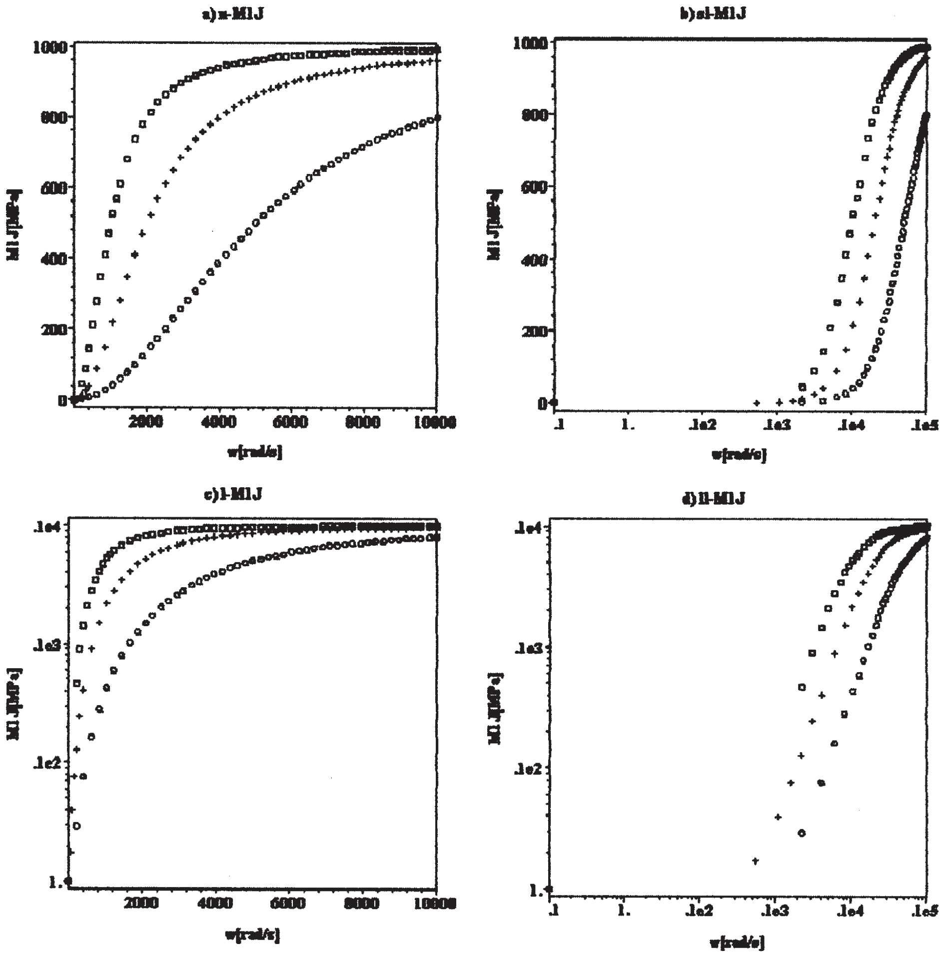

Fig. II.1 Storage modulus, M1J, versus frequency, ω, for different dynamic *retardation* processes - ω_{ret} = 1(□), 2(+), 5(o) [rad/s] – in natural(a), semi-logarithmic(b), logarithmic(c) and double-logarithmic(d) coordinate representations

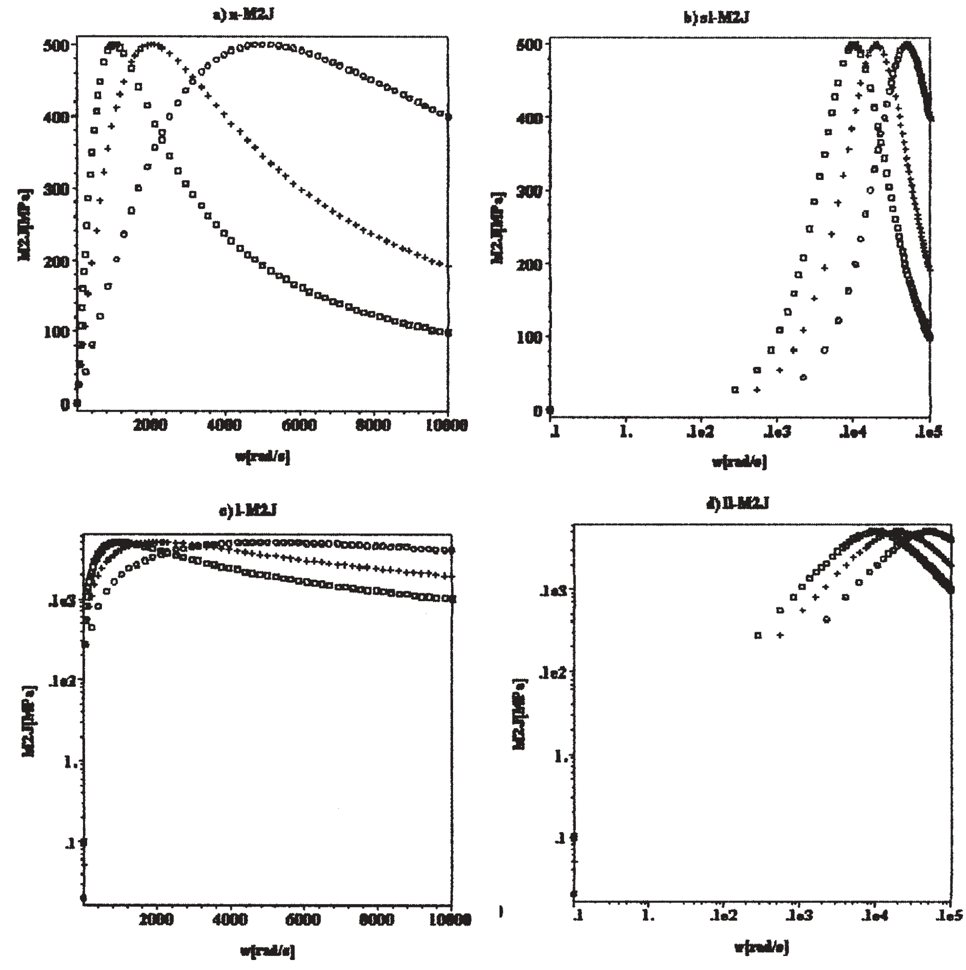

Fig. II.2 Loss modulus, M2J, versus frequency,ω, for different dynamic *retardation* processes - ω_{ret} = 1(□), 2(+), 5(o) [rad/s] - in natural(a), semi-logarithmic(b), logarithmic(c) and double-logarithmic(d) coordinate representations

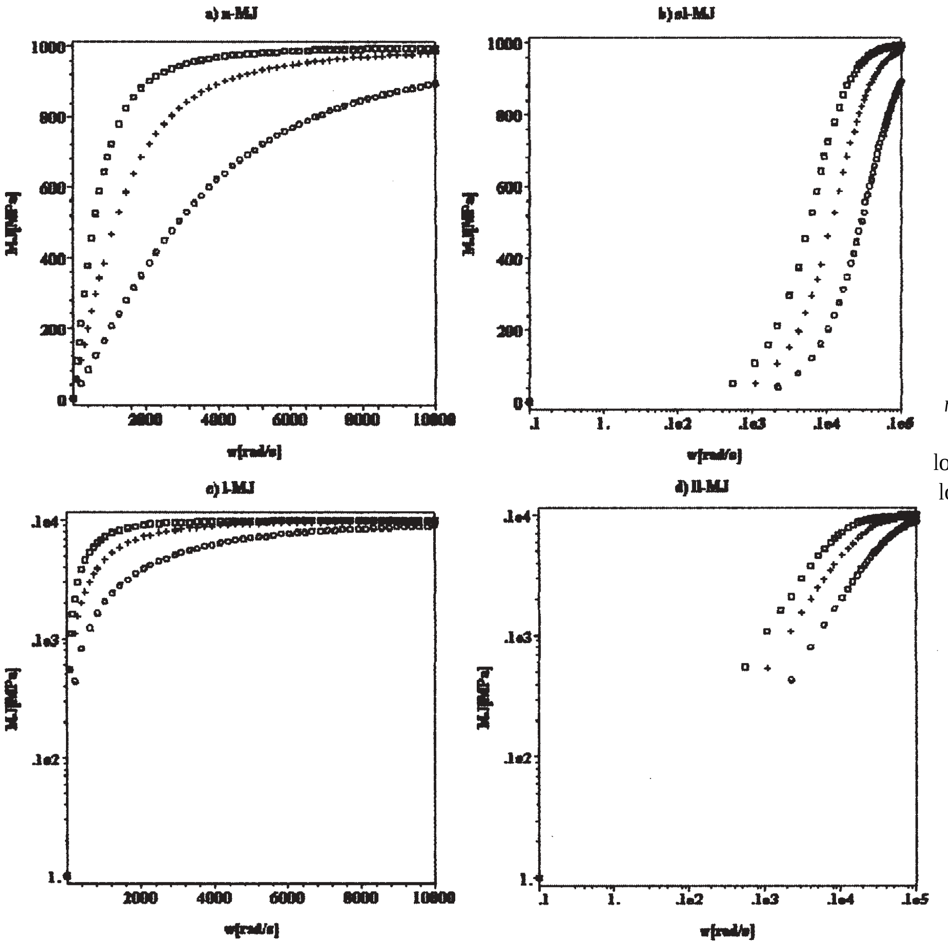

Fig. II.3 Absolute modulus, MJ, versus frequency, ω, for different dynamic *retardation* processes - ω_{ret} = 1(□), 2(+), 5(o) [rad/s] - in natural(a), semi-logarithmic(b), logarithmic(c) and double-logarithmic(d) coordinate representations

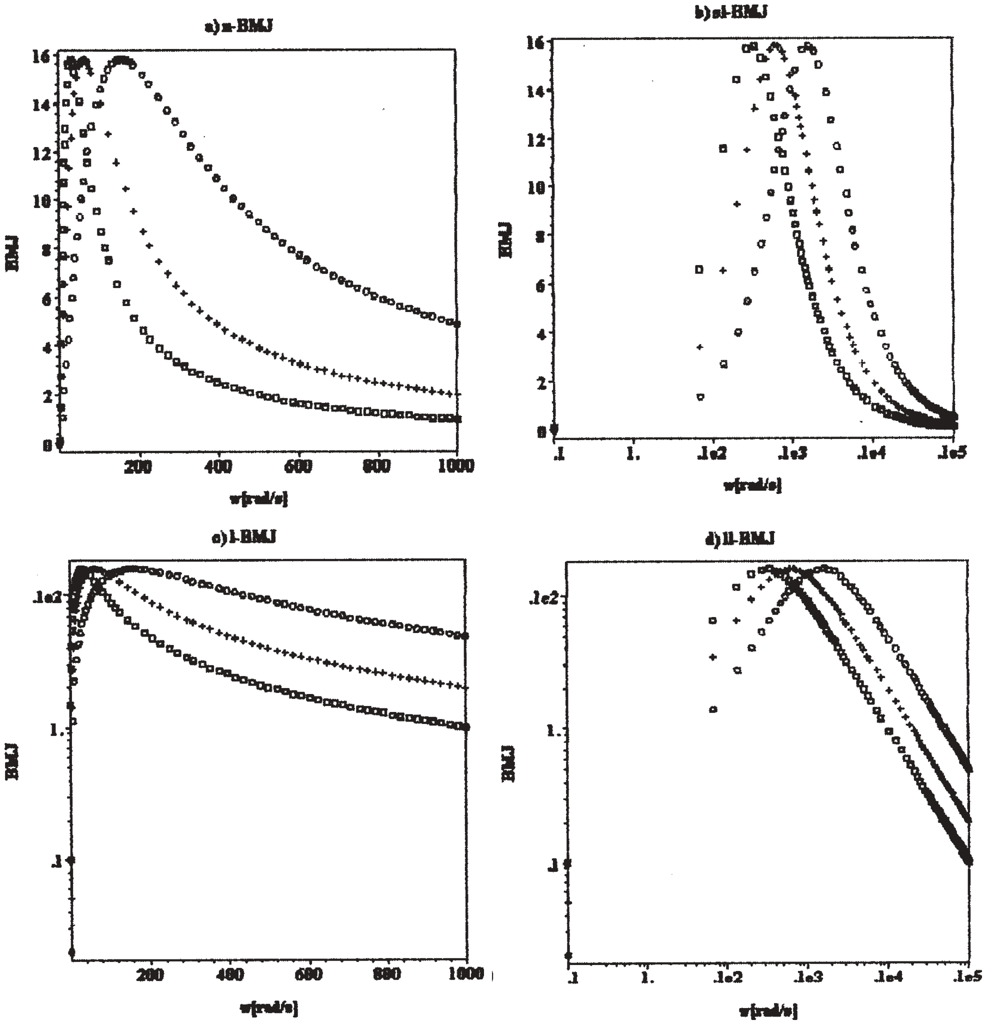

Fig. II.4 Loss factor, BMJ, versus frequency, ω, for different dynamic retardation processes, - ω_{et} = 1(□), 2(+), 5(o) [rad/s] - in natural(a), semi-logarithmic(b), logarithmic(c) and double-logarithmic(d) coordinate representations

Table 2
CALCULATED VALUES OF CHARACTERISTIC FREQUENCIES OF PRIMARY AND SECONDARY RHEODYNAMIC QUANTITIES IN CASE OF σ-CONTROLLED PROCESSES FOR GIVEN VALUES OF RETARDATION FREQUENCIES (W_{RET} = 1, 2, 5 RAD/S)

Viscoelastic Quantity	Characteristic frequencies [rad/s]			
J1	wi{J1}	0.57735	1.154701	2.886751
J2	wm{J2}	1	2	5
	wi{J2}	1.732051	3.464102	8.660255
J	wi{J}	0.707107	1.4142	3.536
BJ	wm{BJ}	31.62277	63.246	158.114
	wi{BJ}	54.77226	109.545	273.861
M1J	wi{M1J}	577.35	1154.701	2886.751
M2J	wm{M2J}	1000	2000	5000
	wi{M2J}	1732.051	3464.102	8660.254
MJ	wi{MJ}	24.02118	48.0423	120.105
BMJ	wm{BMJ}	31.62277	63.246	158.114
	wi{BMJ}	54.77226	109.545	273.861

frequency, to the *high limit* value, M'_∞, at "*infinite*" frequency; on the other hand, (fig. II.2), the loss modulus shows a *maximum, peak* value at intermediate frequency, corresponding to natural ε - frequency of dynamic relaxation process, while zero value corresponds both to *zero* and "*infinite*" frequency. For other *secondary* rheodynamic quantities distinct trends are evidenced - the absolute modulus, (fig. II.3), shows a frequency dependence similar to that of storage modulus, with inflection at lower frequency than in case of storage modulus, while the corresponding loss factor, (fig. II.4), shows a somewhat similitude with that of the loss modulus, even if the maximum and inflection frequencies are at lower values.

Conclusions

In the case of *stress - controlled dynamic retardation processes* the set of σ-characteristic frequencies defined in terms of model rheological parameters is obtained, providing *complete information* as regards the spectrum of linear viscoelastic behaviour.

The *secondary* rheodynamic quantities - including in case of σ-*controlled* processes, on the hand, the storage and loss moduli, as well as, on the other hand, the absolute modulus and the corresponding loss factor, respectively - result on the basis of compliance-like ones, specific features being revealed by using the multiple criterioum of characteristic maximum and/or inflection points.

Given the essential justification of usefulness of the set of characteristic frequencies in case of σ - *controlled dynamic retardation processes*, thoroughly attention is to be considered jointly for consistent characterization and appropriate evaluation of polymeric materials undergoing dynamic stress-controlled conditions of testing.

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