Determination of Damping Coefficients for Sandwich Bars with Polypropylene Honeycomb Core and the Exterior Layers Reinforced with Metal Fabric

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We deduced the equation of motion for free vibrations using Mohr-Maxwell method, considering that the bar is requested to bending by the inertial forces and by the damping forces. We experimentally obtained the dynamic response for sandwich bars with polypropylene honeycomb core, which have the exterior layers made of epoxy resin reinforced with steel fabric. In addition, we determined the damping coefficients per unit mass and per unit length of the bar, for bar with core thicknesses equal to 10, 15 , 20 mm; width of 40 and 60 mm, and the free length of embedded bar equal to 200, 250, 300 and 350 mm. For each type of bar studied, we determined the relationships of exponential type for the calculus damping coefficients (mentioned above), according to the free length of the embedded bar.

Keywords: sandwich beam, damping factor, polypropylene honeycomb, free vibrations

The composite plates and bars could be analyzed using a lot of theories that mostly differ by the inclusions or neglecting the effects of angular deformation and respectively, the rotational inertia.

A theory (First – order Shear Deformation Theory – FSDT) has been developed in [1] and later modified in [2]. This theory relies on a linear distribution of the shear stresses and it requests a correction factor similar with the one from isotropic plates. This theory states that a straight line normal on the median plane before deformation, remains straight without keeping the perpendicularity during deformation (on the median surface).

Exact theories rely on a non-linear distribution of shear stresses along the thickness of the plate or bar. The inclusion of high order terms implies the inclusion of supplementary unknowns. Moreover, when fulfilling both the distribution of shear stresses in thickness is parabolic and if the limit conditions are accomplished on external surfaces, it is not necessary a correction factor. Based on this fact, it was developed a theory [3] (High – order Shear Deformation Theory – HSDT) where it is assumed that stresses and strains normal to the median plane are null. Another theory, in which there are also considered the stresses normal to the median plane, has also been developed in [4, 5] by removing a series of contradictions appearing in previous theories by accepting non linear factors of shear stresses in thickness; also, they did not neglect a part of the normal stresses obtained by the loading of the composite structure.

A much better description can be obtained by using of Layer - Wise Models (LWM). The first attempts to consider each layer in a sandwich structures as a separate bar were made in [6, 7]. In [8], it is expanded the idea for laminated plates by considering each layer an individual Reissner – Mindlin plate. The papers [9] and [10] treated each layer separately using in – plane displacements linear in the thickness direction z. In [11], it is treated each layer separately, by employing a higher – order displacement field for flexural wave propagation analysis in laminated plates.

In [12], a Reissner's mixed variational equation is employed to derive the differential equations, in terms of the introduced stress and displacement variables, that give the dynamic equilibrium and compatibility of each layer. A numerical investigation has been conducted for free – vibration response of cross – ply laminated, simply supported, thick and thin plates for which closed – form solutions are given.

There have also been made some studies on the damped vibrations of Euler – Bernoulli and Timoshenko bar. Relevant to this works are the studies [13, 14]. The material was assumed to be incompressible whereby the same viscoelastic operators could be both used for the flexural and shear deformations. This permitted the use of the normal modes and their orthogonality conditions to solve this viscoelastic forced vibration problem. In [13], it is analyzed a damped and axially loaded Timoshenko bar for random transverse load. Only a special case of damping in the transverse and rotatory motion was considered which allowed, then, using the orthogonality conditions of the undamped modes to decouple the modal equations. In [14], it is obtained the „closed – form" solution, but for an incomplete differential equation of a simply – supported bar with external damping.

In [15], it is presented a general modal approach to solve the linear vibration problem of a uniform Timoshenko bar with external transverse and rotatory viscous damping and different viscoelastic damping in the flexural and shear deformations. With this approach, the bars with boundary conditions can be as conveniently analyzed as a bar with simple supports.

In [16], the dynamic response of a general class of continuous linear vibrating systems is analyzed which passes damping properties close to those resulting in classical (uncoupled) normal modes. First, conditions are given for the existence of classical modes of vibrations in

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a continuous linear system, with special attention being paid to the boundary conditions. Regular perturbation expansions in terms of undamped mode shapes are utilized for analyzing the eigenproblem as well as the vibration response of almost classically damped systems.

The purpose of research done in [17] was to investigate the curvature, and the face / core debonding influence on the vibration behaviour of curved composite sandwich beams built from carbon / epoxy laminate skins over a foam polyurethane core. The sandwich-type beams, flat and curved, with debonding, were prepared by keeping the arc length of the sandwich beams, equal to the length of flat sandwich beams. Natural frequencies and damping loss factor, for sandwich beams, were determined using impulse frequency response technique under free-free boundary conditions.

Recent applications have shown that honeycomb panels from polymer, reinforced with fiber, can be used for new construction or for restoration of existing structures. In [18], there are studied the vibrations of sandwich structures with honeycomb which have the core geometry of sinusoidal type. It was developed a higher order vibration model for studying the vibrations, made by energy methods.

In [19], there were studied the free vibrations of the curved sandwich beams, with flexible core, in different conditions of temperature. The external surfaces and the core of the beam were considered as being made of materials with mechanical properties dependent on temperature. It was shown that the frequency of free vibrations of the beams decreases when the temperature increases.

Theoretical aspects

Equations of motion for transversal vibrations of visco-elastic bars with constant section and external damping are given in [15]. It shows that if the ratio of length and thickness of a bar is greater than ten, then the difference between Timoshenko and Euler-Bernoulli theories for the bending moment, shear force and the medium fiber deformation, are smaller than five percent (5%). It shows that the damping influence of rotational motion of the bar section can be neglected (for the first eigenmodes of vibration). Equations and similar conclusions for bars made of composite materials are presented in [20-21].

To determine the equation of motion, in case of free transversal vibrations of a bar embedded at one end and free at the other, it may be used Mohr-Maxwell method. Thus, if the bar has constant section, the arrow in section x = a is determined by the relationship:

$$w(a,t)=\frac{1}{\langle EI\rangle}\int_0^L M(x,t)\cdot m(x)dx, \qquad (1)$$

where

- L is the bar length;
- $\langle EI\rangle$ is the bar stiffness and it is determined with the relation

$$\langle EI\rangle=\iint_{(S)} E\cdot y^2 dS\,; \qquad (2)$$

- E(x, y) is Young's modulus for bar material;
- m(x) is the bending moment produced by a force equal to the unit applied in the section x=a

$$m(x)=\begin{cases}0, \; for\; x<a\\ -(x-a),\; for\; x\geq a\end{cases}; \qquad (3)$$

- M(x,t) is the bending moment produced by the exterior testings of the bar.

We consider that the bar loading is given by:

- the inertial forces $f_i(x,t)=-\langle \rho A\rangle\cdot \ddot{w}(x,t)$; (4)
- the damping forces $f_a(x,t)=-c^*\cdot \dot{w}(x,t)$, (5)

where

- $\langle \rho A\rangle=\iint_{(S)}\rho(x,y)dS$; (6)
- $\rho(x,y)$ is the density of bar material;
- c^* is the damping coefficient per unit length of the bar.

In these conditions, it occurs:

$$M(x,t)=\int_0^x (x-\tau)\left(\langle \rho A\rangle\cdot \ddot{w}(\tau,t)+c^*\cdot \dot{w}(\tau,t)\right)d\tau. \qquad (7)$$

The solution for motion equation (1) is determined using the method of separating variables:

$$w(a,t)=u(t)\cdot v(a). \qquad (8)$$

The boundary conditions require the following restrictions for the function $v(a)$:

$$\begin{matrix} v(L)=0 & v''(0)=0\\ v'(L)=0 & v'''(0)=0.\end{matrix} \qquad (9)$$

Damping determination is made based on the study of free vibrations produced by an initial deformation of the bar, i.e. the initial conditions:

$$w(a,0)=f(a);\quad \dot{w}(a,0)=0. \qquad (10)$$

The bar vibration has the expression:

$$w(a,t)=\sum_{n=1}^{\infty}u_n(t)\cdot v_n(a), \qquad (11)$$

where $v_n(a)=c_n\left[S''\left(\frac{\beta_n}{L}a\right)-\frac{S(\beta_n)}{S'''(\beta_n)}\cdot S'\left(\frac{\beta_n}{L}a\right)\right]$, (12)

with:

- $S(x)=\frac{1}{2}(chx+\cos x)$; (13)
- β_n are the solutions of equation $ch\beta\,.\cos\beta+1=0$; (14)
- c_n are determined from the orthogonality conditions

$$\int_0^L v_n(x)\cdot v_k(x)dx=\begin{cases}1,\; if\; n=k\\ 0,\; if\; n\neq k\end{cases}; \qquad (15)$$

$$u_n(t)=p_n\cdot e^{-\frac{c}{2}t}\left(\frac{c}{2\omega_n}\sin\omega_n t+\cos\omega_n t\right) \qquad (16)$$

with:

- $c=\frac{c^*}{\langle \rho A\rangle}$ is the damping coefficient per unit mass of the bar;
- $p_n=\int_0^L f(x)\cdot v_n(x)dx$; (17)
- ω_n are eigenpulsations of the bar.

Experimental recording of the free vibrations gives the posibility of damping determination, as follows:

-there are determined the values at which the arrow is zero (the points where the graph intersects the time axis);

-it is determined the period of cancellation of the movement, more precisely, T is the time interval double, between two successive cancellations;

-it is determined the frequency $\nu = 1/T$ and the pulsation $\omega = 2\pi / T$;

-it is determined the damping coefficient

$$\mu = \frac{c}{2} = \frac{1}{KT}\ln\frac{A_i}{A_{i+k}}, \qquad (18)$$

where A_i and A_{i+1} are maximums separated by periods.

Experimental measurements

We built the plates from composite materials with polypropylene honeycomb core (honeycomb which has the thickness with the values , and). The exterior layers of the plates were made †of epoxy resin reinforced with steel fabric.

For these plates, we collected six sets of samples with length equal with and width equal with and respectively .

These were noted as follows:

-the set of samples 1: l = 40 mm; g = 10 mm;
-the set of samples 2: l = 50 mm; g = 10 mm;
-the set of samples 3:l = 40 mm; g = 15 mm ;
-the set of samples 4: l = 50 mm; g = 15 mm;
-the set of samples 5:l = 40 mm; g = 20mm ;
-the set of samples 6:l = 50 mm; g = 20mm .

A sample with the core thickness equal to 20 mm (the core is made from honeycomb of polypropylene), is illustrated in figure 1.

We also considered several variants of embedded of plates, on various lengths, as follows (we will refer to the free portions of the plates - namely the parts on which the accelerometers are located, and on which the measurements will be made; it will be considered schematization from figure 3):

-Variant I: the free length is 200 mm;
-Variant II: the free length is 250 mm;
-Variant III: the free length is 300 mm;
-Variant IV: the free length is 350 mm.

The measuring points are according to the scheme from figure 1 (the points where the accelerometer was positioned) and they will be numbered with P_1 and P_2. Each measurement for each point was made twice.

The data record for a bar of set 1, which has the free length equal to 200 mm and the measurement is made in point P_1, is shown in figure 2.

The processing of these data record and the calculus of the damping coefficient, for a five number of cycles is shown in figure 5. In this processing, based on relation (18), we determined the half from value of damping coefficient per unit mass of the bar.

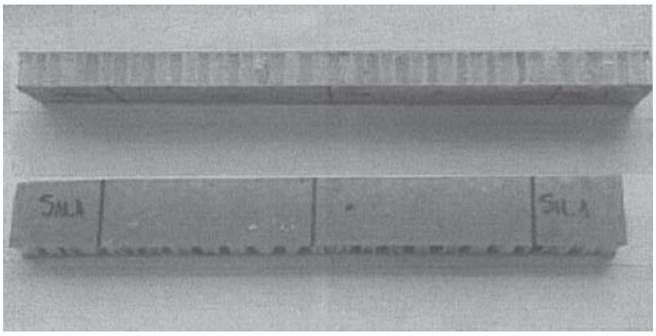

Fig. 1.

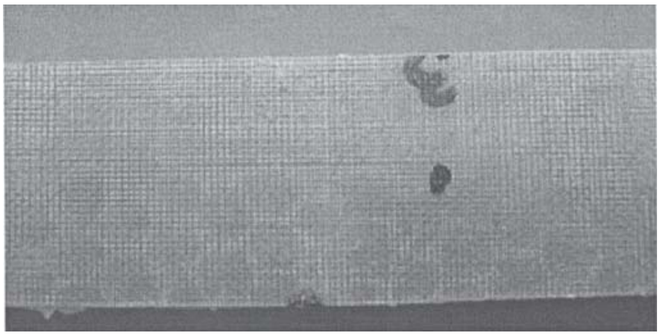

Fig. 2.

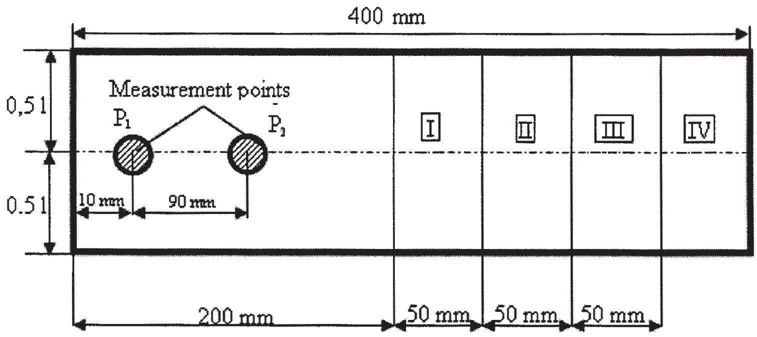

Fig. 3.

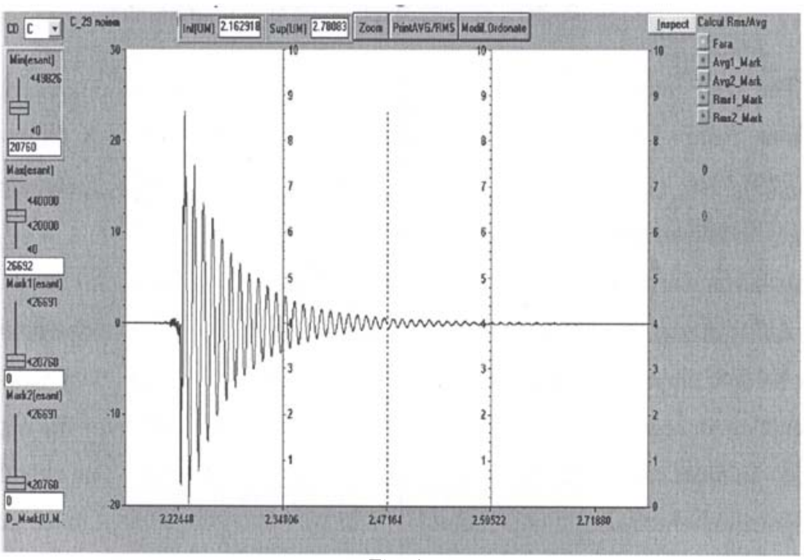

Fig. 4.

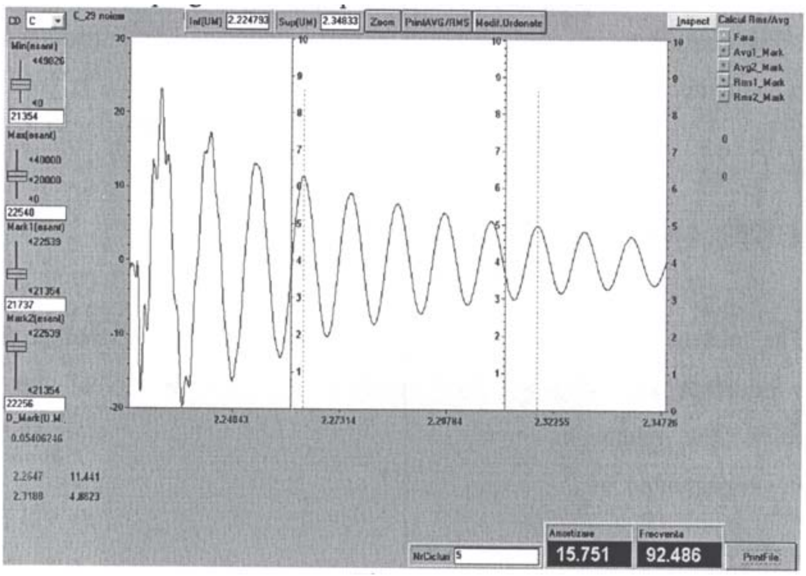

Fig. 5.

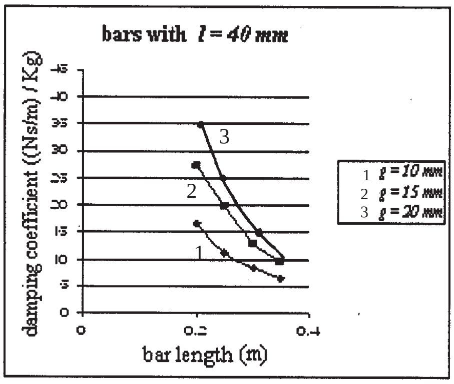

Fig. 6.

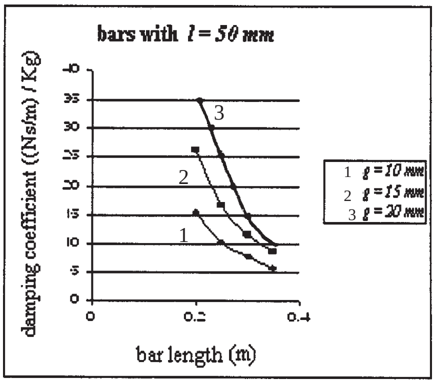

Fig. 7.

Table 1

The set of bars	Mass per unit length of the bar $\langle \rho A \rangle$ (Kg/m)	The formula for the coefficient μ $((Ns/m)/Kg)$	Correlation coefficient (R^2)
1	0.185	$\mu = 55.371 \cdot e^{-6,2122 \cdot L}$	0.9909
2	0.236	$\mu = 54.445 \cdot e^{-6,4656 \cdot L}$	0.9938
3	0.2	$\mu = 115.68 \cdot e^{-7,1736 \cdot L}$	0.9957
4	0.251	$\mu = 110.9 \cdot e^{-7,4098 \cdot L}$	0.9923
5	0.21	$\mu = 183.36 \cdot e^{-7,958 \cdot L}$	0.9954
6	0.272	$\mu = 189.82 \cdot e^{-8,3054 \cdot L}$	0.9832

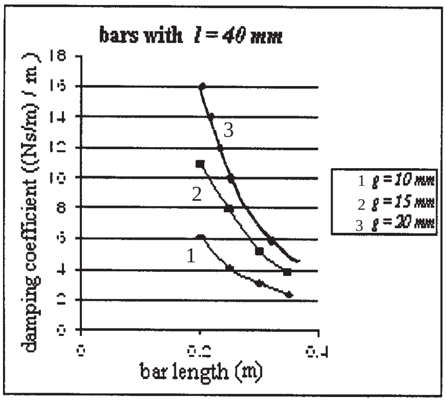

Fig. 8.

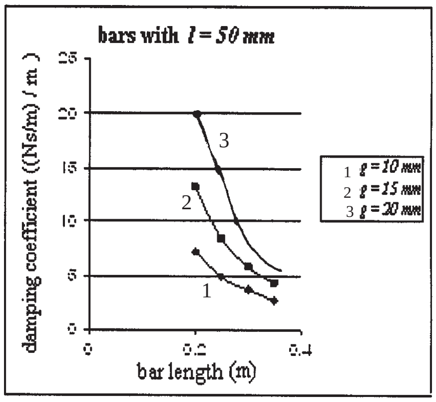

Fig. 9.

Because we have not noticed the significant differences given by the point of measurement, we have made for each set of bars, the average of values of the damping coefficient μ (half of the damping coefficient per unit mass of the bar) for the eight measurements.

The variation of μ coefficient, depending on the free length of bar, for bars which have the width equal with 40 and 50 mm is shown in figures 6 and 7.

We have presented the variation for coefficient μ, because this is directly determined from the processing of experimental records.

In table 1 we present the mass properties and the calculus relationships obtained for coefficient μ, depending on the free length of embedded bar, for the 6 sets of bars.

The damping coefficient per unit length of the bar (c*) is determined with the relation (18).

In figures 8 and 9 is illustrated the damping coefficient variation, per unit length of the bar depending on the free length of the bar (for bars which have width equal to 40 and 50 mm).

Conclusions

The values analysis of damping coefficients indicates that these coefficients must be experimentally determined for each type of material and sample, being difficult to deduce a quantitative correspondence with the parameters which influence the damping, directly or indirectly. The values of damping coefficients may depend on several factors such as: sample dimensions, specific mass or the quantity of material from sample, elastic and damping properties of component materials.

The sample width can influence the damping coefficient by the fact that it determines the surface in which the air friction is acting on the sample. The sample mass or specific linear mass has an influence on the damping coefficient so that, for the samples with higher mass and width, the deformation energy which is stored in the sample through the initial deformation, is dissipated in a larger quantity of material. An influence may occur due to the sample rigidity, explained by the fact that a force initially applied on the sample produces a less deformation if the rigidity is higher.

A good damping of vibrations is achieved in the case in which the composite materials of the external layers have the damping capacity and superior elastic properties. But the influence of these layers is dependent on the interaction with the middle layer and, for this reason, it is difficult to be analytically analyzed.

In addition to these general considerations, we can distinguish the following conclusions:

-both for damping coefficient per mass unit and as well for the damping coefficient per unit length of the bar, the highest values were obtained for bars with core thickness equal with 20 mm and the lowest values were obtained for bars with core thickness equal with 10 mm; therefore, the damping coefficient increases at once with the core thickness of bar;

-we have not noticed the significant differences of the damping coefficient per mass unit of the bar, for the bars which have the width equal with 40 mm, and, respectively, for those which have the width equal with 50 mm; however, the bars which have the width equal with 50 mm have the damping coefficient per unit length of the bar higher than the bars with the width equal with 40 mm;

-both values of the damping coefficient, per unit mass and per unit length of the bar, decrease proportional with free length of the bar, having a dependence of exponential type.

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References

1. YANG, P.C., NORRIS, C.H., STAVSKY, Y., Elastic Wave Propagation in Heterogeneous Plates, Int. Jour. Solids. Struct., 2, 1965, p. 664-684.
2. WHITNEY, J.M., PAGANO, N.Y., Shear Deformation in Heterogeneous Anisotropic Plates, Jour. Appl. Mech., 37, 1970, p. 1031-1036.
3. REDDY, J.N., A Review of Refined Theories of Laminated Composites Plates, Shock and Vibration, 22, 1990, p. 3-17.
4. LIBRESCU, L., Formulation of an Elastodynamic Theory of Laminated Shear Deformable Flat Panels, Journ. Sound and Vibr., 147 (2), 1989, p. 1-12.
5. LIBRESCU, L., KHDEIR, A.A., Analysis of Symmetric Crass-Ply Laminated Elastic Plates Using a High-Order Theory, Part. I: State of Stress and Displacement, Composites Structures, 79, 1990, p. 189
6.KAO, J., RASS, R.J., Bending of multilayer sandwich beams, American Institute of Aeronautic and Astronautics Journal, vol. 6, 1968, p. 1583-
7.SWIFT, G.W., HELLER, R.A., Layered beams analysis, ASCE Journal of Engineering Mechanics Division, vol. 101, 1974, p. 267-282.
8.SEIDE, P., An approximate theory for the bending of laminated plates, Mechanics Today, vol. 5, 1980, p. 451-466.
9. REDDY, J.N., A generalization of two-dimensional theories of laminated composites plates, Communication in Applied Numerical Methods, vol. 3, 1987, p. 173-180.
10. NOSIER, A., KAPANIA, R.K., REDDY, J.N., Free vibration analysis of laminated plates using a Lay-Wise Theory, American Institute of Aeronautic and Astronautics Journal, vol. 31, 1993, p. 2335-2346.
11. VALISETTY, R.R., REHFIELD, L.W., Application of ply level analysis to flexure wave vibration, Journal of Sound and Vibration, vol. 126, 1988, p. 183-194.
12. CARRERA, E., Layer-Wise mixed models for accurate vibrations analysis of multilayered plates, ASME Journal of Applied Mechanics, vol. 65, 1998, p. 820-828.
13. BANERJEE, J.R., KENNEDY, D., Response of an axially loaded Timoshenko beam to random loads, Journal of Sound and Vibration, vol. 101(4), 1985, p. 481-487.
14. ELISHAKOFF, I., LIVSHITS, D., Some closed-form solutions in random vibration of Bresse-Timoshenko beams, Probabilistic Engineering Mechanics, vol. 4(1), 1989, p. 49-54.
15. SINGH, M., ABDELNASER, A.S., Random vibrations of externally damped viscoelastic Timoshenko beams with general boundary conditions, ASME Journal of Applied Mechanics, vol. 60(1), 1993, p. 149-156.
16. NATSIAVAS, S., BECK, J.L., Almost classically damped continuous linear systems, ASME Journal of Applied Mechanics, vol. 65(4), 1998, p. 1022-1031.
17. BUKET, O.B., SRINIVASA, T., An experimental investigation of free vibration response of curved sandwich beam with face/core debond. Journal of Reinforced Plastics and Composites, 29(21), 2010, p. 3208-3218 (2010), DOI: 10.1177/0731684410369721.
18. XUPING, H., PIZHONG, Q., WEI, F., Vibration analysis of honeycomb FRP sandwich beams by a High Order Theory. Conference Proceeding Paper, Earth & Space 2008: Engineering, Science, Construction and Operations in Challenging Environments, 2008, p. 1-11, DOI: 10.1061/40988(323)47.
19. ALASHTI, R A., KASHIRI, N., The effect of temperature variation on the free vibration of a simply supported curved sandwich beam with a flexible core. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 225(3), 2011, p. 537-547, DOI: 10.1243/09544062JMES2429.
20. BOLCU, D., STANESCU, M.M., CIUCA, I., JIGA, G., GHEORGHIU, H., IANCULUI, D., Mat. Plast., **45**, no. 2, 2008, p. 137
21. STANESCU, M.M., BOLCU, D., PASTRAMA, S.D., CIUCA, I., MANEA, I., BACIU, F., Mat. Plast., **47**, no. 4, 2010, p. 492

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