Experimental Determination of the Flow Rate Coefficient Required to Calculate the Flow Rate in the Feeding Zone of a Single Screw Extruder

COSMIN V. JINESCU*
Polytechnic University of Bucharest, Faculty of Mechanics and Mecatronics, Process Equipment Department, 313 Splaiul Independentei, 030138, Bucharest, Romania

The flow rate coefficient was calculated both theoretically, based on calculus relationships, and experimentally, based on data obtained on two extruders with screw diameters of 45 mm and 90 mm, respectively. The concordance of the theoretical results with those obtained experimentally in two situations were verified, namely: free discharge, no backpressure, and discharge with backpressure.

Keywords: flow rate coefficient; feeding zone; single screw extruder; flow rate

The flow rate of an extruder is directly proportional with the flow rate coefficient which can be determined experimentally or calculated theoretically. It is important that the theoretical way of calculus of the flow rate coefficient to be verified by the values obtained experimentally.

To determine experimentally the flow rate coefficient was used a feeding zone with free discharge, as shown in figure 1, consisting of screw 1, barrel 2, feeding hoper 3 with electromagnetic vibrating device 4, shaft 5 with elastic element 6 amd discharge box 7.

The electromagnetic vibrating device 4 when in function vibrates the granular material near the entrance into the screw channel. The rotation frequencies were 8; 12; 15; 20 and 40 rot/min. The following working regimes were used:

-feeding the granular material without using the electromagnetic vibrating device;

-feeding the granular material using the electromagnetic vibrating device.

The amplitude of the vibrations generated by the electromagnetic vibrating device is 2.5 mm at a frequency of 35 Hz. The feeding hoper was positioned in two situations:

-with vertical wall on the left side (fig. 1,a);

-with vertical wall on the right (fig. 1,b).

The form and the characteristic dimensions of the two screws used are shown in figure 2 for the screw diameter of 45 mm and of 90 mm, respectively, whereas in figure 3 is shown the picture of the two experimental set-ups.

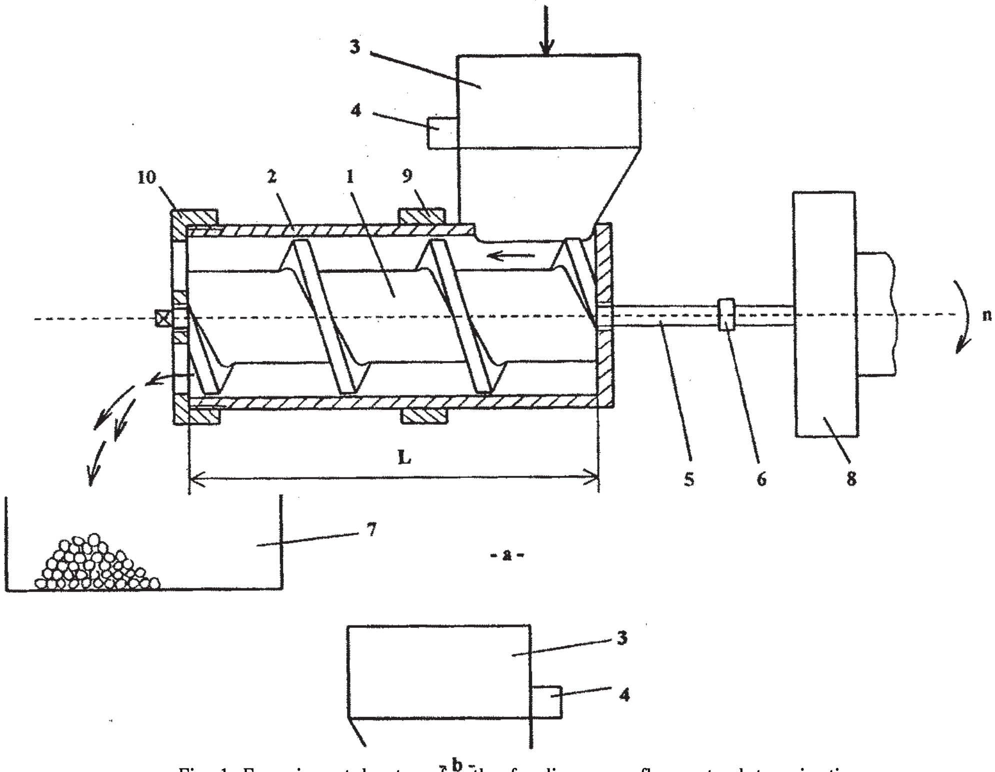
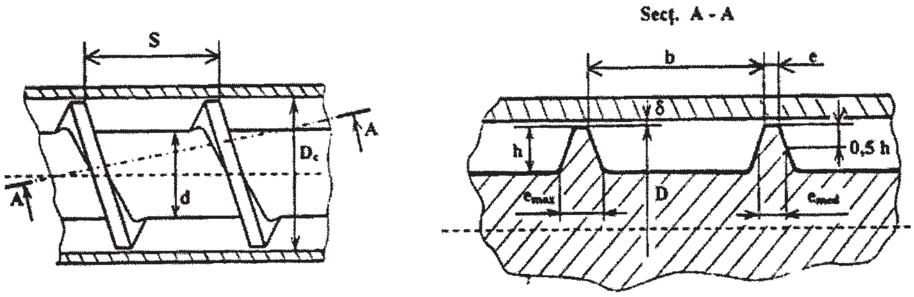


Fig. 1. Experimental set-up for the feeding zone flow rate determination:
- with the vertical wall on the left(fig.1, a); - with the vertical wall on the right (fig.1, b).

* Tel.: 0722655133



D	45	90
L	135	270
S	44	88
d	22	54
h	11,5	18
b	38	77
e	5	9
e_{med}	5	12
e_{max}	5	15
δ	0,1	0,25

Dimensions in mm

Fig. 2. Characteristic dimensions and their respective values for the screws used in experiments

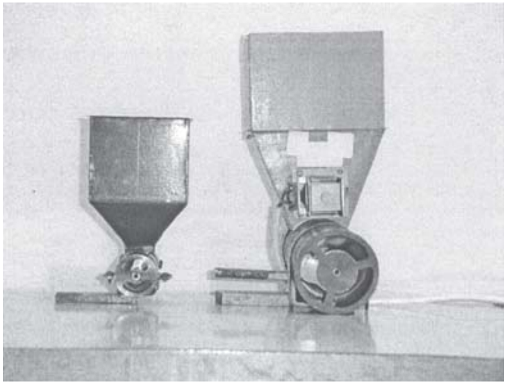


Fig. 3. Picture of the two experimental set-ups

Calculation of the flow rate coefficient

The masic flowrate of the feeding zone was calculated with the relationship:

$$G_m = \frac{M_{gr}}{t}$$

where M_{gr} is the mass of the discharged granules during time t. For D=90mm, t=30”, while for D=45mm, t=60”.

The flow rate per screw rotation was calculated with relationship:

$$G_{m,1} = \frac{G_m}{n} = \frac{M_{gr}}{t \cdot n} \quad (1)$$

where $[G_{m,1}]$=kg/rot, is the flow rate obtained over one rotation of the screw, and n - is the number of rotations during time t. First the dependence of flow rate $G_{m,1}$ on the turation was verified. The experiments were carried out on the screw with the diameter of D=90mm, using HDPE, at four different turations. The results are shown in table 1.

Analysing these results that practically the turation does not influence the flow rate per rotation in both analyzed situations: without vibration and with the vibrating of the granular material. The difference between the average value and the extreme values resulted during experiments is less than 1% (in absolute value) in both cases, which can be assimilated with the admissible experimental errors. As a result, the following experimental work was carried out for only a single turation, n=8rot/min. The results obtained for the two screws and the two granular materials used are shown in table 2 and table 3.

Table 1
FLOW RATE PER ROTATION FUNCTION OF SCREW ROTATION FOR A SCREW OF D=90 mm, USING HDPE GRANULES

n, $\frac{rot}{min}$		8	12,5	20	40	$(G_{m,1})_{med}$ g/rot
$G_{m,1}$ g/rot	Without vibration	118.40	119.28	118.71	118.92	118.83
	Vibrated	123.82	123.91	124.6	124.22	124.14

Table 2
FLOW RATE PER ROTATION FOR A SCREW OF D=45 mm

		$G_{m,1}$, g/rot						$(G_{m,1})_{med}$
Experiment nr.		1	2	3	4	5	6	g/rot
HDPE	Without vibration	18.0	18.5	18.4	18.6	18.1	18.5	18.35
	Vibrated	18.6	18.9	18.7	18.8	18.4	18.85	18.708
PP	Without vibration	19.3	19.75	19.85	19.9	19.9	19.7	19.733
	Vibrated	20.5	20.6	20.6	20.7	20.5	20.7	20.60

Table 3
FLOW RATE PER ROTATION FOR A SCREW OF D=90 mm

		$G_{m,1}$, g/rot					$(G_{m,1})_{med}$
Experiment nr.		1	2	3	4	5	g/rot
HDPE	Without vibration	118.8	118.2	117.2	119	118.4	118.32
	Vibrated	123.9	123.8	123.6	123.4	123	123.54
PP	Without vibration	124.6	124.16	124.2	125	125.1	124.61
	Vibrated	125.6	125.8	126.4	125.6	126.2	125.92

Table 4
BULK DENSITY OF USED GRANULAR MATERIALS

Granular material	Granules' shape	Density, ρ, kg/m^3	Bulk density, ρ_v, kg/m^3	Void fraction, ε_g, %
HDPE	cilindrical	962	512.5	46.7
PP	lenticular	900	540	40

Experimental part

Calculation of the flow rate coefficients

Based on the experimental data one can calculate the value of the experimental flowrate coefficient, $k_{G,exp}$, depending on the volume per rotation of granular material pushed by the screw, $V_{1,tr}$, and the volume of screw channel corresponding to one rotation, $V_{1,cl}$:

$$k_{G,\exp} = \frac{V_{1,tr}}{V_{1,cl}} \quad (2)$$

where:

$$V_{1,tr} = \frac{G_{m,1}}{\rho_v}$$

with ρ_v bulk density of granular material and:

$$V_{1,cl} = \frac{\pi}{4}\left(D^2 - d^2\right)\cdot(S - s) \quad (3)$$

D- barrel internal diameter;
d - screw diameter;
S - hellicoidal barrier flight;
s = e/cos ϕ.
From the above relationships results:

$$k_{G,\exp} = \frac{G_{m,1}/\rho_v}{\frac{\pi}{4}\left(D^2 - d^2\right)\cdot(S - s)} \quad (4)$$

The bulk density ρ_v was determined by weighing of a certain volume of granular material (table 4).

Using the data from tables 2 and 3 and considering the geometry of the tested feeding zones (fig. 1 and 2) the values of $k_{G,exp}$, for two cases were calculated:
– feeding hoper without vibration;
– vibrated feeding hoper.

The feeding hoper (*3*, fig. 1) was tested in the two cases shown in figure 1, *a* and 1, *b* in both situations , without vibrations and vibrated. For the case shown in figure 1, *a* the flowrates were higher with up to 4% compared to those obtained in the case from figure 1, *b*.

For the screw with the diameter *D*=45 mm, from relationship (3):

$$V_{1,cl} = \frac{\pi}{4}\left(45^2 - 22^2\right)\times 10^{-6}\times(44 - 5.3)\times 10^{-3} =$$
$$= 46.84\times 10^{-6}\,\text{m}^3$$

which is introduced in rel. (4) and results:

$$k_{G,\exp} = \frac{G_{m,1}}{46.84\times 10^{-6}\cdot\rho_v} = \begin{cases} 41.6\left(G_{m,1}\right)_{med} \text{ - for HDPE} \\ 39.5\left(G_{m,1}\right)_{med} \text{ - for PP} \end{cases}$$

After the replacement of $(G_{m,1})_{med}$ with its values from table 2, in the case without vibration, results:

$$k_{G,\exp} = \begin{cases} 41.6\times 18.35\times 10^{-3} = 0.7634 \text{ - for HDPE} \\ 39.5\times 19.733\times 10^{-3} = 0.7794 \text{ - for PP} \end{cases}$$

For th screw of diameter *D*=90 mm, from relationships (3) and (4) results:

$$V_{1,cl} = \frac{\pi}{4}\left(90^2 - 54^2\right)\times 10^{-6}\times(88 - 13.5)\times 10^{-3} =$$
$$= 303.33\times 10^{-6}\,\text{m}^3.$$

$$k_{G,\exp} = \frac{G_{m,1}}{303.33\cdot 10^{-6}\cdot\rho_v} = \begin{cases} 6.4\cdot\left(G_{m,1}\right)_{med} - \text{for HDPE} \\ 6.1\cdot\left(G_{m,1}\right)_{med} - \text{for PP} \end{cases}$$

After the replacement of $(G_{m,1})_{med}$ with its values from table 3, in the case without vibration, results:

$$k_{G,\exp} = \begin{cases} 6.4\times 118.32\times 10^{-3} = 0.7572 - \text{for HDPE} \\ 6.1\times 124.61\times 10^{-3} = 0.7601 - \text{for PP} \end{cases}$$

Comparison of the theoretical results

The verifying of the correctitude of the calculation of the flowrate coefficient k_G [1,3,4] with the values determined experimentally, $k_{G,exp}$, is done in two cases:
– free discharge, no backpressure;
– with backpressure.

The flow rate coefficient depends on the friction coefficients between the granular material and the screw, and the granular material and the barrel, respectively, on two directions, and also on the pressure gradient along the screw.

Free discharge case

For the calculation of k_G one needs to know the value of the β angle [2]. This is calculated for the free discharge case, without backpressure, (p_{yb}=0), and results:

$$\tan\beta = \frac{K_2\cdot h_1\cdot\frac{d\overline{p}_z}{dz}}{K_1\cdot h_1\cdot\frac{d\overline{p}_x}{dx}} = \frac{K_2\cdot\frac{d\overline{p}_z}{dz}}{K_1\cdot\frac{d\overline{p}_x}{dx}} \quad (5)$$

From the comparison of the pressure gradients it results:

$$\frac{d\overline{p}_x}{dx} = \frac{\frac{d\overline{p}_z}{dz}}{k\cdot\tan\overline{\varphi}}$$

so that:

$$\tan\beta = \frac{K_2}{K_1}\cdot k\cdot\tan\overline{\varphi} \quad (6)$$

a. For the extruder with diameter D=90 mm and h_1=18mm,

$$K_1 = 1 - \frac{h_1}{2r_1} = 0.8177$$

$$K_2 = 1 - \frac{h_1}{2r_2} = 0.9608$$

where:

$$r_1 = 0.5\cdot D\left(1 + \tan^2\varphi\right) = 49.36\text{ mm}$$
$$r_2 = 0.5\cdot D\left(1 + \tan^2(90 - \varphi)\right) = 229.42\text{ mm}$$
$$\tan\varphi = \frac{S}{\pi\cdot D} = \frac{88}{\pi\cdot 90} = 0.3112$$

from which φ=19.208° .

Also, one calculates:

$$\tan\bar{\varphi} = \frac{S}{\pi(D-h)} = \frac{88}{\pi(90-18)} = 0.389$$

from which φ=23.62°.

The coefficient for the lateral transmittance of pressure k=0.4 . With these data, from relationship (6) results:

$$\tan\beta = \frac{0.9608}{0.8177}\times 0.4\times 0.389 = 0.1828$$

from where β=11.513°.

With value of β so calculated and considering

$$\tan\varphi_s = \frac{S}{\pi(D-2h_1)} = \frac{88}{\pi(90-2\times 18)} = 0.51873$$

from where φ_s=30.463° and φ_e=φ=19.208° it results [1]:

$$k_{w1} = \frac{\cos\varphi_s\cdot\cos(\beta+\varphi_s)}{\cos\beta} = \frac{\cos 30.463\cdot\cos 41.976}{\cos 11.5133} = \frac{0.88768\times 0.7904}{0.9837} = 0.7132$$

$$k_{w2} = 0.5\times\left[1+\frac{\cos\varphi_e\cdot\cos(\varphi_e+\beta)}{\cos\varphi_s\cdot\cos(\varphi_s+\beta)}\right] = 0.5\times\left[1+\frac{\cos 19.208\cdot\cos(19.208+11.5133)}{\cos 30.46\cdot\cos(30.46+11.5133)}\right] = 0.5\times\left[1+\frac{0.9548\times 0.8858}{0.8877\cdot 0.7904}\right] = 1.1027$$

so that,

$$k_G = k_{w1}\cdot k_{w2} = 0.7132\times 1.1027 = 0.7864$$

b. For the extruder with screw diameter D=45 mm and h_1=11.5 mm, it results, using the same relationships as above, the following:

$$\tan\varphi = \frac{S}{\pi\cdot D} = \frac{44}{\pi\cdot 45} = 0.3112$$

from where φ=19.208°.

$$r_1 = 0.5\cdot D(1+\tan^2\varphi) = 0.5\times 45\times(1+\tan^2 19.208) = 24.68\ \text{mm}$$

$$r_2 = 0.5\cdot D(1+\tan^2(90-\varphi)) = 0.5\times 45\times(1+\tan^2(90-19.208)) = 114.72\ \text{mm}$$

$$K_1 = 1-\frac{h_1}{2r_1} = 1-\frac{11.5}{24.68} = 0.534$$

$$K_2 = 1-\frac{h_1}{2r_2} = 1-\frac{11.5}{114.72} = 0.8998$$

With k=0.4 and

$$\tan\bar{\varphi} = \frac{S}{\pi(D-h_1)} = \frac{44}{\pi(45-11.5)} = 0.418 \text{ results:}$$

$$\tan\beta = \frac{K_2}{K_1}\cdot k\cdot\tan\bar{\varphi} = \frac{0.8998}{0.534}\times 0.4\times 0.418 = 0.28179$$

from where β=17.486° .

One also calculates:

$$\tan\varphi_s = \frac{S}{\pi(D-2h_1)} = \frac{44}{\pi(45-2\times 11.5)} = 0.63662$$

from which φ_s=36.09°.

The values obtained are introduced in the relationships for k_{w1} and k_{w2} [1]:

$$k_{w1} = \frac{\cos\varphi_s\cdot\cos(\beta+\varphi_s)}{\cos\beta} = \frac{\cos 36.09\cdot\cos(17.486+36.09)}{\cos 17.486} = \frac{0.8436\times 0.6663}{0.9625} = 0.584$$

$$k_{w2} = 0.5\times\left[1+\frac{\cos\varphi_e\cdot\cos(\varphi_e+\beta)}{\cos\varphi_s\cdot\cos(\varphi_s+\beta)}\right] = 0.5\times\left[1+\frac{\cos 19.208\cdot\cos(19.208+17.486)}{\cos 36.09\cdot\cos(36.09+17.486)}\right] = 0.5\times\left[1+\frac{0.9548\times 0.8384}{0.8436\times 0.6663}\right] = 1.2121$$

$$k_G = k_{w1}\cdot k_{w2} = 0.584\times 1.2121 \approx 0.708$$

One notices that in the case of free discharge the calculated values are :

$$k_G = \begin{cases} 0.708 - \text{for } D = 45\ \text{mm} \\ 0.7864 - \text{for } D = 90\ \text{mm} \end{cases}$$

and the experimental values are:

$$k_G = \begin{cases} 0.7634\div 0.7794 - \text{for } D = 45\ \text{mm} \\ 0.7572\div 0.7606 - \text{for } D = 90\ \text{mm} \end{cases}$$

which means a non-concordance with the theoretical results, in absolute value, under 10% in case of screw diameter D=45 mm and under 4% for D=90 mm.

Backpressure case

An extruder with the characteristic dimensions corresponding to the one with screw diameter D=90 mm (fig. 2). The granular material is characterized through: coefficient for the lateral transmittance of pressure k=0.40, friction coefficient between barrel and granular material [5] $f_{cx}=f_{cz}=f_c=0.28$. In this case dp_{yb} / dL=20 MPa/m (usual value).

Besides the above mentioned values there are also calculated:

$$K_3 = 1-\frac{h_1}{r_1} = 1-\frac{18}{49.359} = 0.6353$$

$$K_4 = 1-\frac{h_1}{r_2} = 1-\frac{18}{229.42} = 0.92154.$$

After replacements results β=0.1185, from where β=7.51°. After replacements in the relationships for k_{w1} and k_{w1} [1] results for k_G,

$$k_{w1} = 0.769$$

$$k_{w2} = 1.09366$$

$$k_G = k_{w1}\cdot k_{w2} = 0.80876$$

Using the experimental data from literature the following values for the flow rate coefficient have been obtained:

$$k_{G,\exp} = \begin{cases} 0.70\div 0.74 & - \text{for PE[6]} \\ 0.75 & - \text{for PU[7]} \\ 0.857\div 0.905 & - \text{for LDPE, HDPE, PP[8]} \end{cases}$$

It is noticed that the calculated value for the flow rate coefficient (k_G=0.80876) is comprised with a tolerance of $\pm$ 10% in the interval of experimental values from literature, respectively k_G=0.70 ÷0.905.

Conclusions

In the case of free discharge, without backpressure, it was noticed a deviation of the flow rate coefficient values obtained experimentally compared to those calculated (absolute value) of less than 10% for the extruder screw diameter D=45 mm and of less than 4% for the screw diameter D = 90mm. For the backpressure case, at the usual value of 20 MPa/m, for the extruder screw diameter of 90 mm, resulted that the theoretical values for the flow rate coefficient are in concordance with those obtained experimentally [6-8] with a deviation (absolute value) of maximum 10%.

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