The Resistance to Fracture Determination for Composite Plates with Random Distribution of Reinforcement

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In this paper we introduce a formula for the calculus of resistance to fracture for the composite plates with a random reinforcement's distribution. Also, we give a formula for the calculus of upper limit of resistance to fracture for the composite plates, described as depending on the volumetric proportion of reinforcement. As applications, we give the variation of resistance to fracture depending on the volumetric proportion for two types of composite materials and, in addition, we determine the characteristic curves and the resistance to fracture for a traction test considering the two composite materials.

Keywords: composite materials, strength to rupture, traction

The goal of damage is to predict the response of a material in the presence of damage that initiates at some stress state and generally increases with increasing stress up to macroscopic crack initiation or failure; prediction of the conditions for failure is also a fundamental goal of damage mechanics. In general, damage may be thought of a surface discontinuities (micro cracks) and volume discontinuities (micro voids). Essentially all materials exhibit cracks and voids at some scale; thus the "damage-free" state is a function of the scale of the phenomena under consideration.

The most common types of damage in fibrous composites are fiber breakage, fiber/matrix debonding, matrix cracks, fiber kinking and for large diameter fibers, radial cracks in the fibers. We consider damage that can only increase or remain constant over time; there is no healing.

As damage occurs, the material loses stiffness and exhibits nonlinear, inelastic response with permanent strains after unloading. The inelastic response is the result of sliding friction at damage sites as well as any inelastic response of the constituent phases. Since the process is irreversible, nonlinear analysis techniques must be employed.

The earlier published works on damage mechanics appear to be those by Kachanov [5] and Rabotnov [16] on the application of a continuous damage variable to creep failure of metals. The concept was generalized, within the framework of irreversible thermodynamics, for isotropic materials under multiaxial loads [3, 4, 6, 13-15].

The general damage theory that serves as the foundation for this model was originally proposed in [7]. Later the general damage theory was adapted for application to laminated composites [8]. The theory has been shown to be very robust for predicting the damaged response of composites under a wide variety of conditions. It is based upon the method of local state expressed in terms of state variables and the associated thermodynamic forces. According to [13], "the method of local state postulates that the thermodynamic state of a material medium at a given point and instant is completely defined by the knowledge of the values of a certain number of variables at that instant, which depend only upon the (material) point considered. Since the time derivatives of these variables are not involved in the definition of the state, this hypothesis implies that any evolution can be considered as a succession of equilibrium states." [18].

For the most general configuration, Ladeveze's theory considers a composite to be a laminated structure consisting of two elementary constituents: layers of composite and interfaces that separate the composite layers. The interface is considered to be a mechanical surface connecting two adjacent composite layers; it is included in the model only when delamination is of interest. When delamination is not of interest, the model is an assemblage of composite layers [19, 20].

The theory is called the "mesoscale" composite damage theory because it is based upon the assumption that the damage is uniform through the thickness of individual layers of the composite [21], [22]. A layer is any continuous thickness of plies which have identical fiber orientation. Mesoscale is a term indicating that the scale of the analysis is between micromechanics (i.e., the level of the fiber and the matrix) and laminate analysis. The theory is based upon the mean value of the stress in each layer and allows the damage state to vary from layer to layer in a laminate. When delamination is of interest, damage between layers is introduced through consideration of damage to the idealized interfacial layer, [11, 2]. Additional developments of the model were concerned with the computation of the intensities of the different damage mechanisms up to ultimate fracture [9] and extension of the model to variable temperature ranges [1].

The "mesoscale" composite damage theory assumes that the response of a damaged layer, at any instant of time (load state), can be expressed in terms of elastic moduli degradation and inelastic strains due to damage and/or matrix plasticity. Degradation of the elastic moduli is expressed in terms of damage parameters that are functions of the associated thermodynamic forces which serve as damage evolution parameters. The modulus degradation parameters are internal variables and the thermodynamic forces are the corresponding associated variables (in the thermodynamic sense). The theory includes provisions for coupled damage evolution for the

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multi-axial states of stress that are typically present in laminates, as well as differences in the effects of damage evolution due to tensile and compressive normal stresses. The latter feature allows for difference in material response associated with crack opening and crack closure.

The form of the damage evolution law generally varies with the type of material, reflecting the dependence on the micro structural damage mechanisms. It is expected that the fiber size, microstructure (or morphology) and strength, matrix strength and fiber/matrix interfacial strength all influence damage evolution. The micro-level damage mechanisms are not identified explicitly in the mesoscale damage model; damage evolution is based on experimental observations of the response of a damage layer. In this sense, the theory is phenomenological. The theory allows the damage evolution law to change form during the loading history as new damage mechanisms occur.

The mesoscale composite damage theory has been applied successfully to polymeric, metallic and ceramic matrix composites made with continuous fibers, short fibers or woven fabrics. A more general treatment of the theory and references to other applications can be found in [10].

Experimental part

The theoretical modell for determining the resistence to fracture

To estimate of resistance to fracture of a composite material we considered that fibers have an elastic behavior untill fracture and the matrix has a nonlinear character in case of exceeding maximum deformation for the fibers' fracture. In the case of a unidirectional composite strechted along the fibers, fracture happens when the fibers fail. The practical results indicates that the maximum value of medium strain has lower values than theoretically obtained in the previous hypothesis. The explanation consists in the fact that not all fibers have the same resistance to fracture, some give in and the intact fibers take over the entire stress.

In the case of transversal stretching related to the fibers' direction or shearing, it is considered that fracture resistances coincide with the matrix's resistance to fracture for each type of stress.

In the case of complex stress, when the tensor of tensions has more non zero components, fracture criteria are used. These keep count of the size of each separate tension as well as their correlated effect.

According to the Hill criterion [17], the normal tension of fracture for an unidirectional composite, following a certain direction, is given by the relation:

$$\sigma_r = \frac{1}{\sqrt{\frac{\cos^4\theta}{\sigma_{r,l}^2} + \frac{\sin^4\theta}{\sigma_{r,t}^2} + \cos^2\theta\sin^2\theta\left(\frac{1}{\sigma_{r,lt}^2} - \frac{1}{\sigma_{r,l}^2}\right)}}, \quad (1)$$

in which:

- θ is angle between the considered direction and fibers directions;
- $\sigma_{r,l}$ is the normal tension of fracture in the direction of fibers;
- $\sigma_{r,t}$ is the normal tension of fracture in the transversal direction on fibre;
- $\sigma_{r,lt}$ is the tangential tension of fracture which acts in orthotropy planes of the plate.

In accordance with Hill criterion, the tangential tension of fracture in a non orthotropic plane, can be calculated with the relation [17]:

$$\tau_r = \frac{1}{\sqrt{4\cos^2\theta\sin^2\theta\left[\frac{2}{\sigma_{r,l}^2} + \frac{1}{\sigma_{r,t}^2}\right] + \frac{\left(\cos^2\theta - \sin^2\theta\right)^2}{\sigma_{r,lt}^2}}}. \quad (2)$$

In practice are used plane composite plates which need to have the same resistance properties along any direction in the plate's plane. This fact is possible if:

$$\sigma_r = \sigma_{r,l} = \sigma_{r,t} \quad (3)$$

and

$$\tau_r = \sigma_{r,lt} \quad (4)$$

for any angle θ.

The relations (3) and (4) are satisfied only if:

$$\sigma_r = \sqrt{3}\tau_r. \quad (5)$$

This relation is very restrictive, but we accept by hypothesis that it remains valid for the case in which fibers have a random distribution. We assimilate these plates, from resistance's point of view, with composite plates for which fibers are equally distributed after any direction in the plate's plane.

In this conditions in which plane isotropy is accepted, we consider the tension of fracture $\sigma_{r,c}$ for composite as an average value, obtained by integrating the relation (1) and then applying a medium relation. Thus we have:

$$\sigma_{r,c} = \frac{2}{\pi}\sqrt{\sigma_{r,l}\cdot\sigma_{r,t}}\cdot\int_0^1 \frac{1}{\sqrt{x^4 + \sigma_{r,t}\cdot\sigma_{r,l}\left(\frac{1}{\sigma_{r,lt}^2} - \frac{1}{\sigma_{r,l}^2}\right)x^2 + 1}}dx. \quad (6)$$

Starting from this relation we determine a maximum value on the form (which can't be overtaken by the value of composite's tension of fracture):

$$\sigma_{r,c} < \frac{2}{\pi}\sqrt{\sigma_m\cdot\sigma_f\left[V + (1-V)\frac{E_m}{E_f}\right]}, \quad (7)$$

in which:

- σ_m is the tension of fracture to stretch the matrix;
- σ_f is the tension of fracture to stretch the fibers;
- E_m is the elasticity modulus of matrix;
- E_f is the elasticity modulus of fibers.

In the figure 1 is gived the variation of resistence to fracture for a plate from fireproof phenolic resin reinforced with fiberglass (E_m = 3000 MPa, σ_m = 40 MPa, E_f = 74000 MPa, σ_f = 2500 MPa).

In the figure 2 is given the variation of resistance to fracture for a plate from poliesteric orthoftalic resin reinforced with fiberglass (E_m = 4000 MPa, σ_m = 80 MPa, E_f = 74000 MPa, σ_f = 2500 MPa).

If the volumetric proportion is small (V<0.3), then the properties can be very different from one point to another in the plate's plane. The hypothesis of isotropy isn't valid any more. For this reason, the graphics showing the variation of resistance to fracture are drawn for a volumetric proportion $V \in [0.3, 0.7]$.

Results and discussions

We tested to traction two sets of samples, thus:

- a set of tests for a composite plate from phenolic resin reinforced with fiberglass, having the volumetric proportion of reinforcement V = 0.45 and with the chemical composition presented in figure 3 (sample 1);
- a set of tests for a composite plate from poliesteric resin reinforced with fiberglass, having the volumetric proportion of reinforcement V = 0.55 and with the

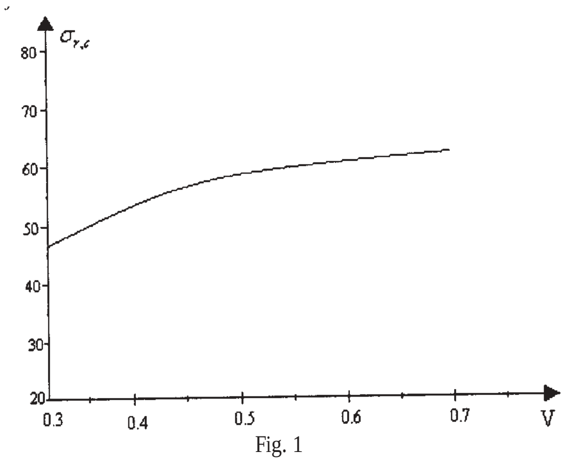

Fig. 1

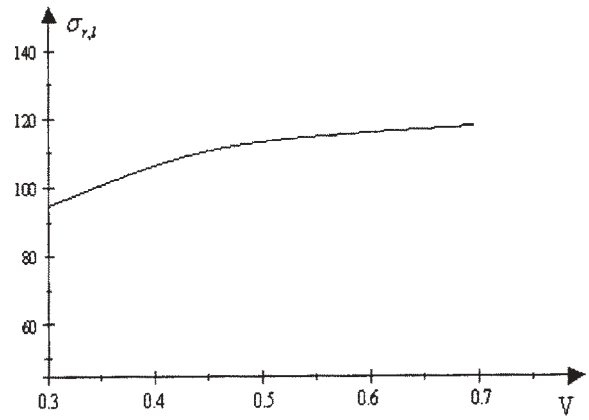

Fig. 2

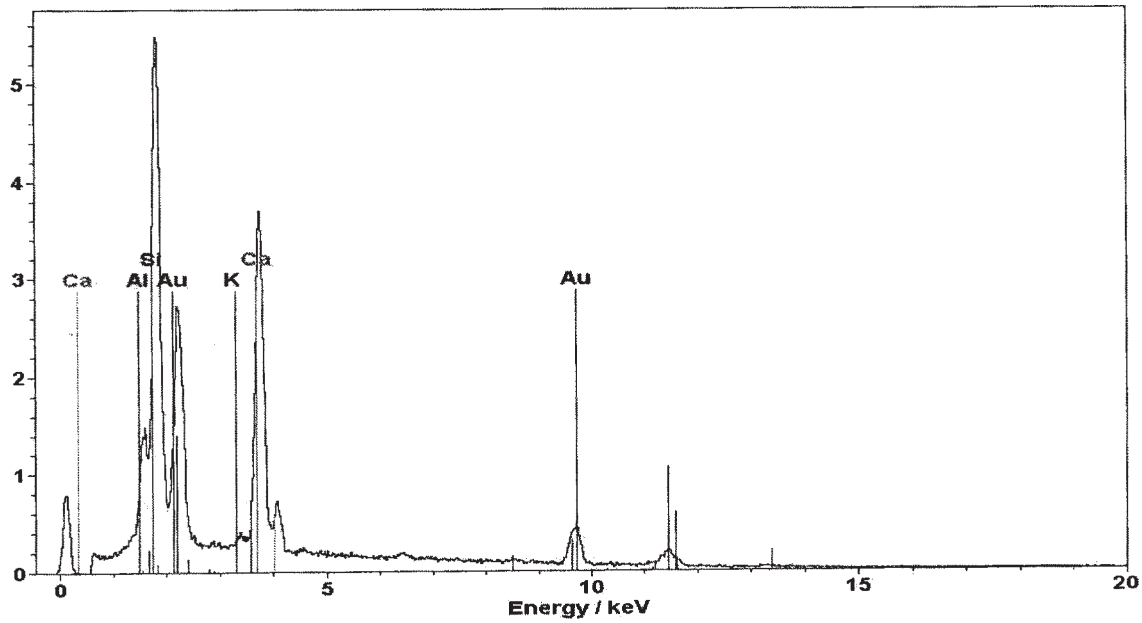

Fig. 3

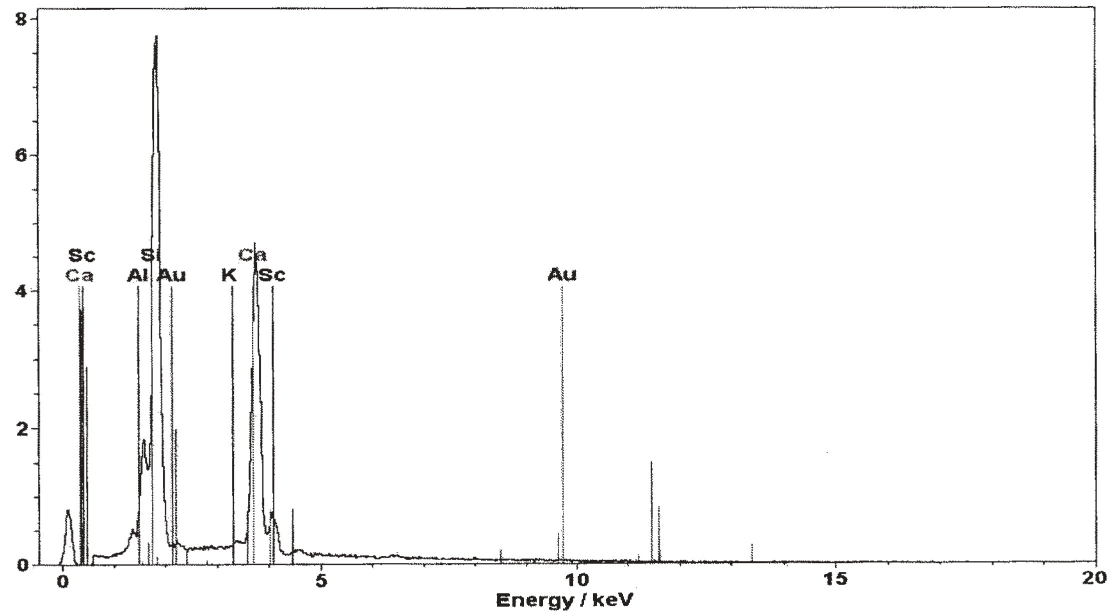

Fig. 4

chemical composition presented in the figure 4 (sample 2).

The test boards used and the way we achieved fracture, are presented in figure 5 (sample 1) and in figure 6 (sample 2).

In figure 7 (sample 1) and in figure 8 (sample 2) the way in which we achieved the fracture is presented in section.

From these figures we notice that fibers do not have a preferential direction of orientation and we can't evidentiate a certain order. Hence, these plates can be associated with plates which have a random distribution

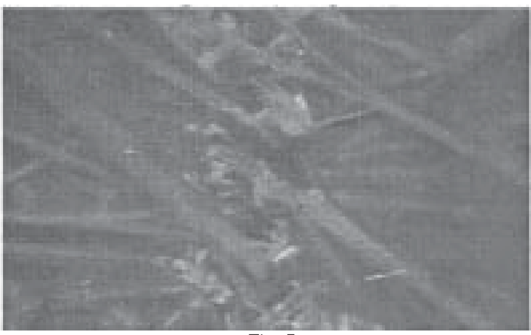

Fig. 5

Fig. 6

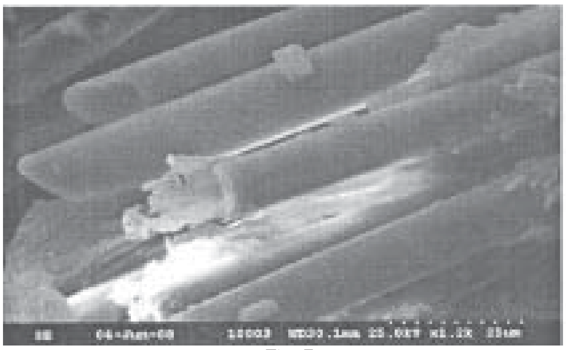

Fig. 7

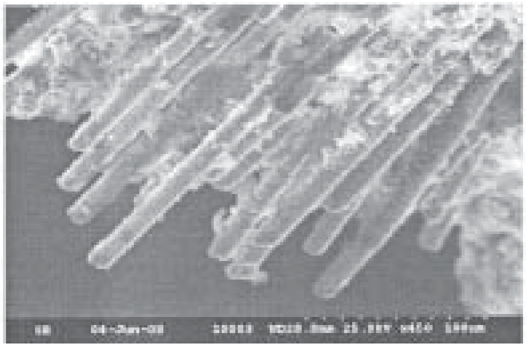

Fig.8

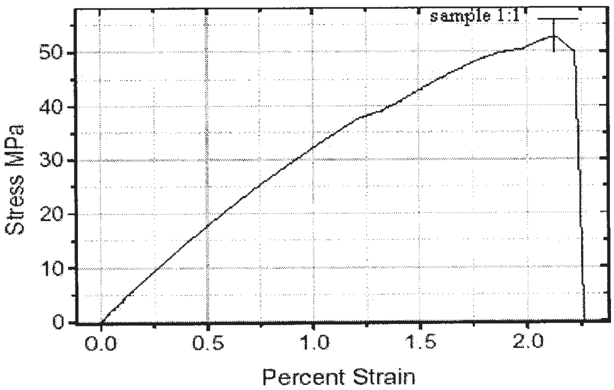

Fig. 9

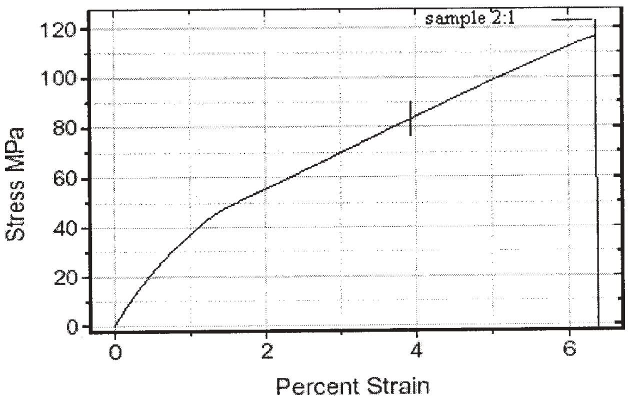

Fig. 10

of reinforcement and thus we consider that we can apply the results obtained using the relation (6).

For the sample 1 in the fracture area is observed the total snatching of matrix from fibers. In the case of sample 2, the adherence among fibers and matrix is better. The matrix also breaks, as shown by the traces of resin in the fracture area.

In figure 9 (sample 1) and in figure 10 (sample 2) we present the characteristic curves for the composite materials of each plate.

On the figures 9 and 10 we notice that the constituent equation has a near linear character.

For the sample 1 we obtained the tension of fracture value of 53 MPa, and for the sample 2 the tension of fracture is 116 MPa.

Conclusions

In the case of composite materials for which the reinforcement has a random distribution, the resistance to fracture is smaller than in unidirectional composites' case. This thing can be explained considering the smaller ammount of fibers that take over the stress with respect to the total number of fibers built in the composite. Moreover, the percentage of woven fibers is smaller than the one in case of unidirectional composites.

Because the composite plates with random distribution of reinforcement can be assimilated as having isotropy in the plate's plane, it results that they will have properties closer to matrix properties. This fact happens because the fibers present a high grade of anisotropy.

The analyse of sections in which fracture took place and the diagrams of characteristic curves show that fractures were sudden, without the existence of a flow area. This fact confirms that damage of composite material appears at the fibers' fracture.

Two types of fracture can appear:

-a type of fracture in which the matrix is detached from the fibers in the fracture area; this is justified by the thing that after breaking, the fibers come off the matrix;

-a type of fracture in which, simultaneous with fibers' fracture, the matrix fractures also,still keeping contact with the fibers in the fracture area.

Theoretically, the resistance to fracture, obtained with the relation (6), for the sample 1 is 55.8 MPa, and for the sample 2 is of 111.4 MPa.

Comparing this with the experimental results we noticed that:

-for the sample 1 the theory gives a greater value for resistance to fracture;

-for the sample 2 the theory gives a lower value.

Both values are compared to the ones experimentally obtained.

We can explain it because for sample 1 the fracture is „clean" (without traces of matrix on the fibers) and for sample 2 stresses are partially taken over by the matrix.

In the case of composite plate with phenolic resin, the specific deformation to fracture is a little over 2% and the fracture is characterized as „fragile" fracture. For the plate with poliesteric resin, the specific deformation to fracture is over 6%, being also present ductile fracture phenomena.

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