The Variation of the Natural Frequencies of Road Vibrator-rollers, as a Function of the Parameters of Neoprene Vibration Isolation Elements

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This paper studies the influence of positioning angles of neoprene vibration isolation elements from the first elastic stage, upon the natural frequencies of the vibrator-roller type CVA-10. A simplified computing dynamic model with four degrees of freedom has been used in order to determine the influence of positioning angle on the natural frequency by means of experimental results.

Keywords: neoprene elements, positioning angle, vibrator-roller, natural frequencies

In their structure, the vibrator-rollers contain neoprene elements, which realize the elastic link between the different subassemblies and generate and maintain the desired vibrations of the roller-compactor, while isolating and reducing the undesired vibrations of the other subassemblies, such as the driver's seat, the engine-hydraulic pump group, and other auxiliary equipment [1]. Neoprene elements have a variety of other vibration isolation applications, e.g., for tram railway supporting or for building base isolation against seismic actions [2,3]. Numerous experimental researches and finite element simulations have studied neoprene elastic properties and their influence on equipment modal behaviour [4-6].

ICECON company has designed a self-propelled vibrator-roller, called CVA-10, made of two stages of vibration isolation elements using rubber dampers. Using two stages instead of a single vibration isolation stage is obviously much more advantageous [7-10].

Angular Frequency Analysis

In order to obtain the desired vibrations of the roller-compactor and to reduce the undesired vibrations of other subassemblies, an influence analysis of the stiffness of the neoprene damping elements is necessary. In this paper, a first part of this analysis is provided, i.e., the influence of the stiffness of the neoprene damping elements on the CVA-10 natural frequency drift [7,8].

Figure 1 shows the self-propelled vibrator-roller CVA-10, composed of the following elements: 1 – roller-compactor; 2 – front chassis; 3 – first vibration isolation stage; 4 – second vibration isolation stage; 5 – rear chassis; 6 – rear roller (used for the rear drive). Taking into account the symmetry of the vibrator-roller with respect to the longitudinal median plane and the symmetrical construction of the front chassis with respect to the roller-compactor axis, the CVA-10 vibrator-roller can be modeled as in figure 2. So, in this paper a 2D vibrator-roller model is used. This 2D simulation model is characterized by 4 degrees of freedom, where the two vibration isolation stages have been modeled by using only elastic elements, due to the fact that the difference with respect to the use of viscoelastic elements is about only 5%, so it can be neglected.

The kinetic energy of the CVA-10 mechanical system, described by the dynamic model in figure 2, is [11-14]:

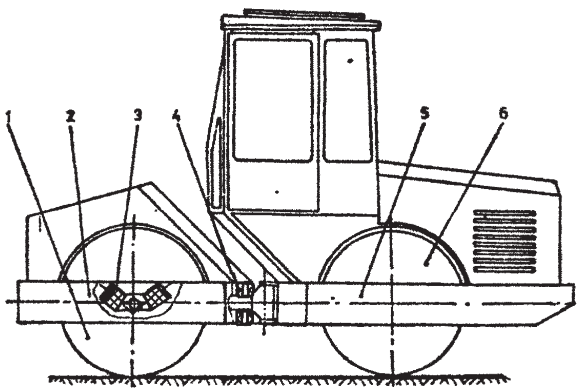

Fig. 1. Self-propelled vibrator-roller CVA-10

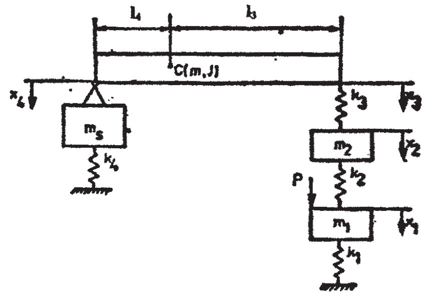

Fig. 2. 2D simulation model of CVA-10

$$2T = m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + m_3\dot{x}_3^2 + m_4\dot{x}_4^2 + 2m_{34}\dot{x}_3\dot{x}_4, \qquad (1)$$

where: $m_1 = m_v$ – mass of the roller-compactor; m_2 – mass of the front chassis; $m_3 = \frac{ml_4^2 + J}{l^2}$ = reduced mass of the rear chassis at supporting point 3; $m_4 = \frac{ml_3^2 + J}{l^2} + m_s$ = reduced mass of the rear chassis at supporting point 4, to which is added the mass m_s of the static drive roller; $m_{34} = \frac{ml_3 l_4 - J}{l^2}$ = reduced mass of the rear chassis; C(m,J) is the mass center; m – the mass and J – mass moment of inertia according with the normal axe passing thru C.

Using the matrix notation and the inner product in Hilbert spaces, the square expression (1) can be written as:

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$$2T = \langle \dot{\mathbf{x}}, \mathbf{M}\dot{\mathbf{x}} \rangle, \qquad (2)$$

where $\dot{\mathbf{x}}$ is the velocities vector, with $\dot{\mathbf{x}}^T = [x_1, x_2, x_3, x_4]$ and where **M** is the inertia matrix.

The inertia matrix **M** is positive definite, symmetric and nonsingular:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & m_{34} \\ 0 & 0 & m_{34} & m_4 \end{bmatrix}. \qquad (3)$$

The deformation potential energy of the mechanical system, with respect to the static equilibrium position, has the following expression [11-14]:

$$2\Pi = k_1 x_1^2 + k_2 (x_1 - x_2)^2 + k_3 (x_2 - x_3)^2 + k_4 x_4^2. \quad (4)$$

The quadratic form (4) can be written in matrix notation as:

$$2\Pi = \langle \mathbf{u}, \mathbf{K}_0 \mathbf{u} \rangle, \qquad (5)$$

where:

$\mathbf{K}_o$ = diag$\{k_1, k_2, k_3, k_4\}$ – the stiffness matrix;

u–the elastic deformation vector, with $\mathbf{u}^T = [u_1, u_2, u_3, u_4]$.

The components of the elastic deformation vector **u** are determined from the displacements x_j = 1,...,4, as follows:

$$u_1 = x_1, \quad u_2 = x_1 - x_2, \quad u_3 = x_2 - x_3, \quad u_4 = x_4.$$

The relation between displacements and deformations is:

$$\mathbf{u} = \mathbf{A}\mathbf{x}, \qquad (6)$$

where:

A – the influence matrix of the displacements on deformations;

x – displacements vector, with $\mathbf{x}^T = [x_1, x_2, x_3, x_4]$.

The matrix **A** can be written as:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (7)$$

Using relation (6), the quadratic form (5) can be expressed in the coordinates of the displacements vector **x** as follows [15]:

$$2\Pi = \langle \mathbf{A}\mathbf{x}, \mathbf{K}_0 \mathbf{u} \rangle. \qquad (8)$$

If one considers the matrix $\mathbf{A} = (a_{ij})$, i, j = 1,...,n as a continuum linear operator on the Hilbert space X^n, then the adjoint operator is $\mathbf{A}^T = (a_{ij})$, i, j = 1, ...,n. Thus, relation (8) becomes [15-17]:

$$2\Pi = \langle \mathbf{x}, \mathbf{A}^T \mathbf{K}_0 \mathbf{u} \rangle, \qquad (9)$$

or, by taking into account relation (6),

$$2\Pi = \langle \mathbf{x}, \mathbf{A}^T \mathbf{K}_0 \mathbf{A}\mathbf{x} \rangle. \qquad (10)$$

By denoting $\mathbf{K} = \mathbf{A}^T\mathbf{K}_o A$, it finally results:

$$2\Pi = \langle \mathbf{x}, \mathbf{K}\mathbf{x} \rangle. \qquad (11)$$

For the elastic case and without considering any disturbing forces, the differential equations of motion are given by the Lagrange equations of second kind, as follows [11-14]:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0. \qquad (12)$$

In order to solve the differential equation (12), one searches the solutions of the following form [18]:

$$\mathbf{x} = \mathrm{Re}(\mathbf{a}e^{ipt}). \qquad (13)$$

where:

x– column displacements vector ($\mathbf{x}^T = [x_1, x_2, x_3, x_4]$);

a-vector of the unknown amplitudes, with $a^T = [a_1, a_2, a_3, a_4]$;

p – natural angular frequency.

By replacing the solution (13) in (12), one obtains the following algebraic equations system [18]:

$$(\mathbf{K} - p^2\mathbf{M})\mathbf{a} = 0. \qquad (14)$$

The algebraic system (14) has to be satisfied by the vector **a** and scalar *p*, thus it is necessary to determine the eigenvalues and eigenvectors of the square matrices **K** and **M**.

The determination of the natural angular frequencies (eigenvalues p_1, p_2, p_3, p_4) and of the eigenvectors was realized using a FORTRAN code. Thus, the 4 eigenmodes were determined, each one corresponding to a different positioning angle of the neoprene damping elements [19].

The natural angular frequencies were computed for different angles between the compression axis of the neoprene damper and the vertical axis. Four different neoprene compounds were considered for the first vibration isolation stage of the self-propelled vibrator-roller CVA-10 [20].

In dynamic regime, the variation of the stiffness coefficient, corresponding to the first vibration isolation stage, with respect to the positioning angle α, is shown in figure 3, being described by the following equation [19,20]:

$$k_\alpha = k_c \cos^2\alpha + k_y \sin^2\alpha, \qquad (15)$$

where:

k_a – equivalent stiffness coefficient corresponding to the positioning angle α;

k_c – equivalent compressive stiffness coefficient, i.e., for α = 0;

k_y – equivalent shear stiffness coefficient, i.e., for α = 90°.

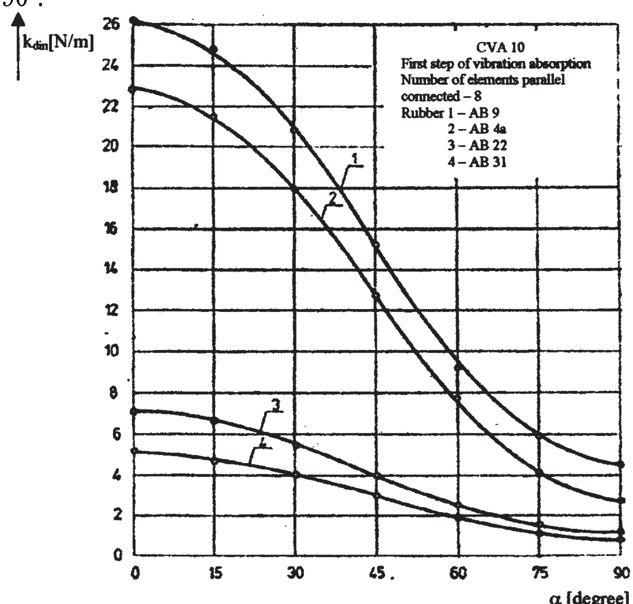

Fig. 3. Variation of the stiffness coefficient of the first vibration isolation stage with respect to the positioning angle α

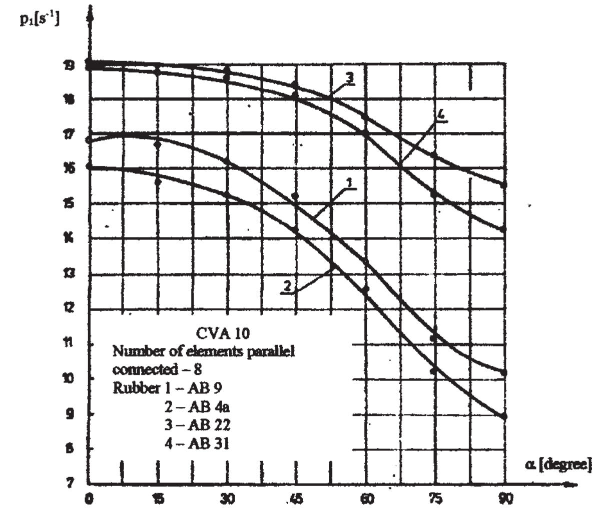

Fig. 4. Variation of the first natural angular frequency p_1 with respect to the positioning angle α and taking into account the quality of the neoprene

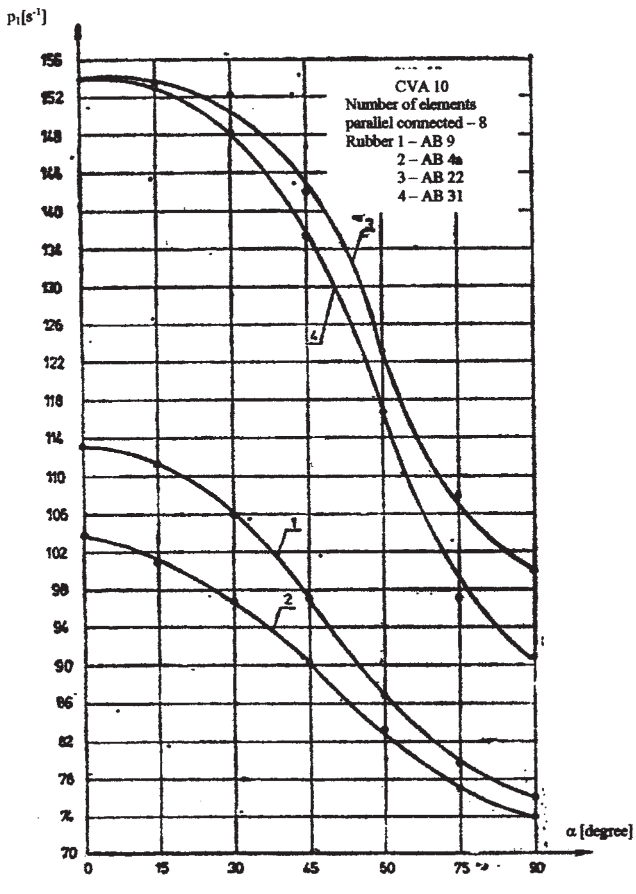

Fig. 5. Variation of the second natural angular frequency p_2 with respect to the positioning angle α and taking into account the quality of the neoprene

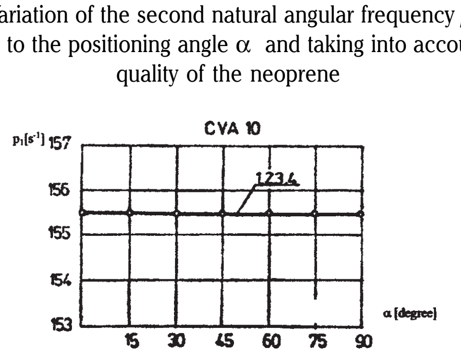

Fig. 6. Variation of the second natural angular frequency p_3 with respect to the positioning angle α and taking into account the quality of the neoprene

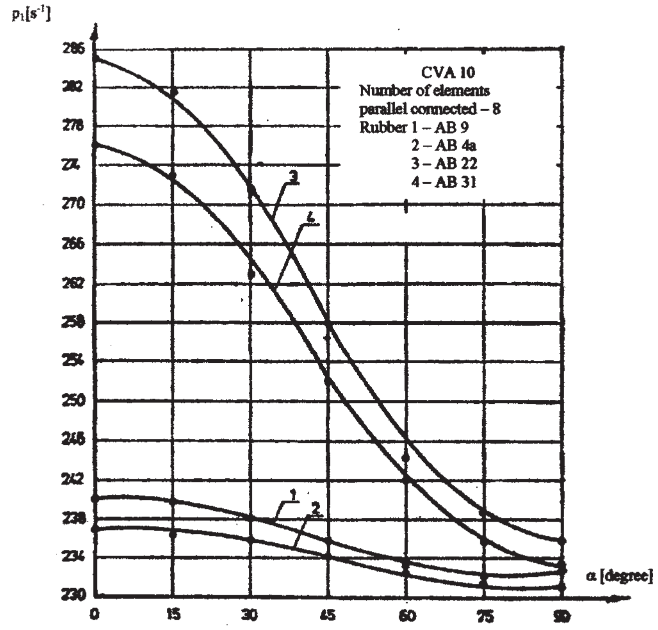

Fig. 7. Variation of the second natural angular frequency p_4 with respect to the positioning angle a and taking into account the quality of the neoprene

Figures 4, 5, 6 and 7 show the variations of the natural angular frequencies, i.e., p_1, p_2, p_3, p_4, with respect to the positioning angle α and taking into account the quality of the neoprene [1,19,20].

Figures 4-7 show that, if the inertia and stiffness characteristics of CVA-10 do not change in time (during the dynamic regime), then for different values of the positioning angle α between 0 and 90°, the natural angular frequencies p_1, p_2 and p_4 are variable, while the third natural angular frequency p_3 remains constant. This variation analysis is very important for the design of the vibrator-roller, in order to find the best technical solutions for the neoprene dampers, in what concerns their functionality, as well as the neoprene compound quality. The goal is to determine the dynamic system to operate in post-resonance, i.e., to have $\omega > p_4$. During the design phase, the appropriate positioning angle α is found taking into account two criteria: 1) the working regime criteria, ensuring the desired vibrations of the roller-compactor and the reduction of the undesired vibrations of other subassemblies; 2) the neoprene elements bearing capacity criteria.

The computation of the natural angular frequencies was performed for the following constant inertia and stiffness characteristics of CVA-10, which have been experimentally determined [8]:

$$k_1 = k_4 = 9 \cdot 10^7 N/m, \quad k_3 = 3{,}04 \cdot 10^6 N/m, \quad k_2 = k_{din}(\alpha)$$
$$m_1 = 1700kg, \; m_2 = 736kg, \; m_3 = 7198kg,$$
$$m_4 = 5154kg, \; m_5 = 3174kg,$$

Conclusions

The variation of the natural angular frequencies p_1, p_2 and p_4 upon the positioning angle α is quite similar with the variation with the equivalent stiffness coefficient of the first elastic stage.

The third natural angular frequency p_3 remains constant, without being influenced by the variations of the positioning angle α.

The values of the natural angular frequencies for α = 0° (neoprene dampers working in compression) are different, depending on the quality of the neoprene. On the other side, for $\alpha = 90^o$ (neoprene dampers working in shear), the natural angular frequencies converge to a single value. This means that, when working in shear, neoprene dampers of the same shape and size, but made of different neoprene compounds, lead to almost the same values of the natural angular frequencies, so in this case it is only the shear strength of the damper criteria that matters.

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