New Relations for the Calculus of Elastical and Mechanical Characteristics of Polyester Composites Reinforced with Randomly Dispersed Fibers

DUMITRU BOLCU [1]*, MARIUS MARINEL STĂNESCU[2], ION CIUCĂ [3], OCTAVIAN TRANTE [3], BAYER MIHAI[4]

[1] University of Craiova, Faculty of Mechanics, 165 Calea Bucureşti, 200620, Craiova, Romania

[2] University of Craiova, 13 A.I. Cuza, Department of Applied Mathematics, 200396, Craiova, Romania

[3] University Politehnica of Bucharest, Faculty of Materials Science and Engineering, 313 Splaiul Independenţei, 060042, Bucharest, Romania

[4] Politehnica University of Bucharest, 313 Splaiul Independenţei, 060021, Bucharest, Romania

Using the linearity of the term $\sqrt{\sigma_l \sigma_t}$ we will deduce a new formula for the calculus of the fracture strength for composites with randomly dispersed fibres. This formula will depend on the reinforcement volume fraction. Using the mean values for longitudinal and/or transversal modulus, with respect to the rotated reference system, we will obtain an original relation for the calculation of the elasticity modulus for composite plates with a randomly dispersed reinforcement. We preferred to write this relation in a similar manner to the one used for the failure strength, using the linearity related to $\sqrt{E_l E_t}$. Experimental determinations were made for three sets of samples obtained from composite plates with various mass fractions of reinforcement, confirming thus the theoretical results obtained by the authors.

Keywords: composite materials, fracture strength, longitudinal/transverse modulus

Depending on the reinforced material or resin used, composite materials allow us to obtain a diversity of mechanical properties. This fact makes it difficult to determine the mechanical characteristics function of the fibres-matrix ratio of the composite. The existing theories outlined the composites as homogeneous, generally anisotropic, where the material constants are obtained in relation to the properties of the constituents. Good results are generally obtained in static problems, but serious deficiencies occur in the case of vibrations, especially due to the attenuation effect which was observed in the case of composite materials.

Only some theories have a solid theoretical background for determining the characteristics of the composite materials. The theoretically obtained results correspond more or less to those obtained experimentally. Irrespective of the theory applied, the results describe the mechanical properties corresponding to the basic physics principles. Therefore, a tensile stress will lead to an elongation following the stress direction, and a uniform compression cannot lead to an expansion/dilatation of the material. We cannot give credibility to any result contradicting even partly these elementary truths.

A frequently used theory is the blends theory, based on an elementary similitude with the blend of gases, where the constituents coexist, each exercising its own partial pressure. With the observance of composite structure, the constituents are presumed heteronymous in space, each one having individual deformations. The laws of the blends can be easily formulated, but the main problem of the application of the theory of the blends in the case of the composite blends is the analytic specification of the interactions of constituents and of the constituent equations for the blend, being known the geometrical distribution and the constituent equations for every individual constituent.

Using the theory of blends in [1] simple relations are determined, which lead to good results for Young's modulus and Poisson's coefficient along the fibres. For the other elastic coefficients it is suggested a relation which has the disadvantage to depend on a parameter which characterizes the interaction fibres-matrix, the geometry of the fibres, their arrangement, and which must be determined empirically. The extreme values for the elastic modulus were determined in [2, 3] in the case of the fibres randomly distributed and with different diameters, but having a given volume fraction.

In [4] is presented a model which enables the calculation of the shear modulus using a mathematical transcription of the interaction fibre-matrix and simultaneously taking into account the differences between the arrangements of the fibres over two different directions of the composite section. This model is used in [5] in order to make a micro-mechanical analysis for the components of the composite and its behaviour. This method has the advantage of generating the hypotheses during the analysis, and the heterogeneous compound is considered as being a homogeneous transversally isotropic one, whose properties can be identified for several fibre-matrix combinations.

For some fibre-reinforced composites, in [6] it is shown the existence of a nonlinear relation among stress-strain. In [7] it is shown that, for the composites epoxy boron-resin or epoxy bleak lead-resin, the nonlinear behaviour is due to the matrix composition that mainly affects the shear module, while the relations stress-strain on the fibres direction, and also on the transverse direction remain almost linear.

Methods of analysis of nonlinear relation of constituents are presented in [8-9].

In [10] it is presented a method for determining the elasticity coefficient taking into account both constitutive equations of the components and conditions for continuity on the separation surfaces for stresses as well as for strains. Starting from the stresses and strains of the constituents, we introduced two matrices which characterise the

* email: bolcu@mecanica.ucv.ro

average values for the stresses and strains of the composite. It is thus determined the whole matrix of the elastic coefficients (not only one coefficient specific to each single test).

If for the composites with a given orientation of the fibres there are theories by means of which the mechanical properties are quite precisely determined, in case of randomly distributed fibres composites, the theoretical results may have informative values, experimental checking being also necessary. A method for determining the fracture strength for a composite plate with a random distribution of the reinforcement fibres is presented in [11]. This method consists in the mediation of the fracture strength when the reference system rotates with a 90° angle. It is thus obtained a relation for the calculus of the fracture strength, which is based on an irrational integral, which depends on the characteristics of the constituents being difficult to use in current applications.

The fracture strength determination is rendered more difficult by the introduction of the material's damage which may be thought of as surface discontinuities (micro cracks), or volume discontinuities (micro voids) [12]. The most common types of damage in fibrous composites are fibre breakage, fibre/matrix debonding, matrix cracks, fibre kinking, and for large diameter fibres, radial cracks in the fibres. The study of these types of damage is made in [13] and [14], where it is presented a general damage theory of the laminated composite materials. The theory is based upon the mean value of the stress in each layer and allows the damage state to vary from layer to layer in a laminate. When delamination is of interest [15, 6]), damage between layers is introduced through consideration of damage to the idealised interfacial layer. In [17], additional developments of the model were concerned with the calculation of the intensities of the different damage mechanisms up to ultimate fracture and extension of the model to variable temperature ranges [18].

In [19] it is shown that the form of the damage evolution law generally varies with the type of material, reflecting the dependence on the micro structural damage mechanisms. It is expected that the fibre size, micro-structure and strength, matrix strength and fibre/matrix interfacial strength all influence damage evolution. The micro-level damage mechanisms are not identified explicitly in the model; damage evolution is based on experimental observations of the response of a damage layer.

In this sense, the theory is phenomenological, it does not allow the damage evolution law to change form during strain when new damage mechanisms occur.

Failure strength

In order to estimate the failure strength of a composite material, we considered that fibres have an elastic behaviour until failure, and the matrix has a nonlinear character in case of exceeding maximum deformation for the fibres failure. In the case of a unidirectional composite subjected to tension along the fibres, it is considered that failure occurs when the fibres fail. The practical results indicate that the maximum value of medium strain has lower values than those theoretically obtained in the previous hypothesis. The explanation consists in the fact that not all fibres have the same failure strength, some give in and the intact fibres take over the entire stress.

In case of transversal tension related to the fibres direction or in case of shear, it is considered that failure strengths coincide with the matrix failure strength for each type of stresses. In case of a complex stress state, when the tensor of tension has several non zero components, failure criteria are used. These take into account the size of each separate tension as well as their correlated effect.

In [11] it is presented a formula for the calculus of the failure strength for composite materials with randomly distributed fibres. This formula has the disadvantage that one should calculate an integral whose value varies according to the properties of the constituents. In [11] it is also presented a relation which is used to determine the maximum value attainable by the failure strength. Starting from this relation, through the linearity depending on $\sqrt{\sigma_l \sigma_t}$ it is determined the following formula for the calculation of the failure strength for composite materials with randomly distributed reinforcement:

$$\sigma_r = \sigma_m(1-V) + V\sqrt{\sigma_t \sigma_l}\ , \qquad (1)$$

where σ_m is the failure strength of the matrix material; V is the volume fraction of the fibres; σ_l is the failure strength along the fibres of the uniderectional composite with V proportion of reinforcement; σ_t is the failure strength in a transversal direction, perpendicular to the fibres direction, of the uniderectional composite with reinforcement volume fraction.

According to [20], it is accepted that:

$$\sigma_t = \sigma_m, \qquad (2)$$

$$\sigma_l = \sigma_f\left[V + (1-V)\frac{E_m}{E_f}\right], \qquad (3)$$

where σ_f is the fibres failure strength; E_m is the matrix elastic modulus; E_f is the fibres elastic modulus.

In figure 1 is presented the variation of the failure strength for a composite with polyesteric resin matrix having, E_m=1800MPa, σ_m = 20MPa, randomly reinforced with fibreglass having E_f = 74000 MPa and σ_f = 2500MPa

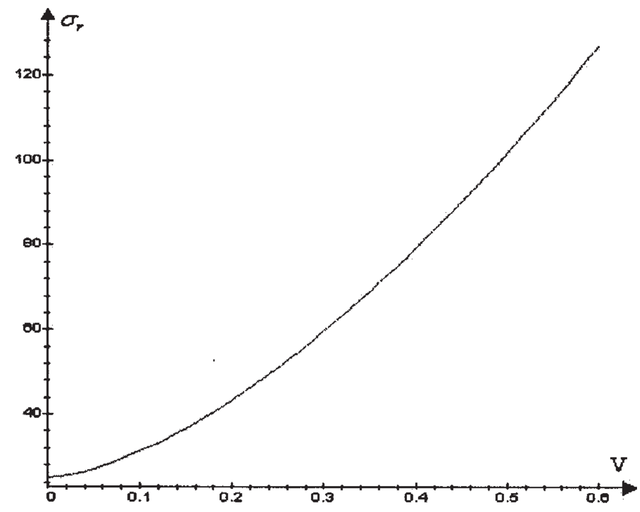

Fig. 1. Variation of the failure strength for a composite with polyesteric resin matrix

The elastic modulus

In the case of unidirectional composite materials, the elastic moduli on two perpendicular axes in the orthotropic plane are calculated with the relations [20]:

$$E_x(\theta) = \frac{1}{\dfrac{\cos^4\theta}{E_l} + \dfrac{\sin^4\theta}{E_t} + \cos^2\theta\sin^2\theta\left(\dfrac{1}{G_{lt}} - 2\dfrac{\upsilon_{tl}}{E_t}\right)} \qquad (4)$$

and

$$E_y(\theta)=\frac{1}{\frac{\sin^4\theta}{E_l}+\frac{\cos^4\theta}{E_t}+\cos^2\theta\sin^2\theta\left(\frac{1}{G_{lt}}-2\frac{\upsilon_{tl}}{E_t}\right)}, \quad (5)$$

where θ is the angle between the considered axes and the orthotropic axes of the unidirectional composite; E_l is the elastic modulus of the unidirectional composite along the fibres; E_t is the elastic modulus of the unidirectional composite perpendicular to the fibres; G_{tl} is the shear modulus of the unidirectional composite; ν_{tl} is the Poisson's ratio.

Similar to the method presented in [11], we obtain an averaging formula for the elasticity modulus in the case of the composite material with randomly dispersed reinforcement:

$$E=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}E_x(\theta)d\theta. \quad (6)$$

Since for θ = 0 we obtain $E_x=E_l$ and $E_y = E_t$ and for θ=π/2 we obtain $E_x=E_t$ and $E_y=E_l$ it results that the elastic modulus can also be calculated with the relation:

$$E=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}E_y(\theta)d\theta. \quad (7)$$

Since the result depends on all the parameters which appear in the relations (5) and (6) and is difficult to use in practice, as in the case of failure strength, using the linearity depending on the term $\sqrt{E_l E_t}$. we obtain:

$$E=E_m(1-V)+V\sqrt{E_t E_l}. \quad (8)$$

For the elasticity moduli along the fibres and in transversal direction on fibre we can use the relations presented in [12]:

$$E_l=E_m(1-V)+E_f V, \quad (9)$$

$$E_t=\frac{E_m E_f}{E_m V+E_f(1-V)}. \quad (10)$$

In figure 2 it is presented the variation of the elasticity modulus for the same type of composite used for the failure strength.

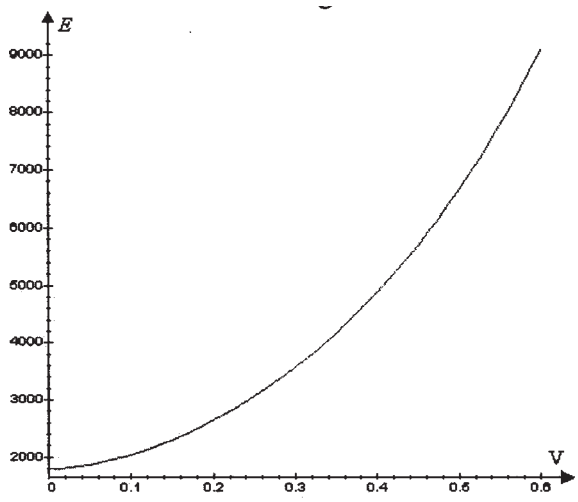

Fig. 2. Variation of the elasticity modulus for the same type of composite used for the failure strength

Experimental part

The experiments were performed on three different sets of samples made of polyester resin reinforced with fibreglass. Table 1 presents the sample characteristics.

The samples were tested to tensile stress until failure. Table 2 presents the test parameters and the main results obtained.

In figure 3 is presented one test-piece from each set of samples which was tested to traction. One can observe the way in which the fibres failed, and the crack propagation in section.

Figures 4, 5 and 6 present the characteristic curves obtained for sample 1, sample 2 and sample 3 respectively.

These diagrams were obtained by determining the deformation and strain for a number of 121 measurements for sample 1; 80 measurements for sample 2, and 61 measurements for sample 3.

Results and discussions

The analysis of the stress-strain curves in figures 4, 5 and 6 indicates an almost linear dependency between strain and deformation. Hooke's law may therefore be accepted as valid for composite materials with randomly distributed reinforcement as well. Consequently, we can experimentally determine the elasticity modulus for the composite material with the formula:

Table 1

Order number	Quantity of fibres on m^2 (gr)	Volumetric proportion of fibres	Test board section area (mm^2)	Test board width (mm)	Distance (mm)
Sample 1	100	0,1	14,51	25	33,4
Sample 2	200	0,15	19,38	25	35,4
Sample 3	450	0,3	23,15	25	37,7

Table 2

Order number	Stretching speed (mm/min)	Maximum strain (KN)	Elongation under maximum strain (mm)	Maximum elongation (mm)	Failure strength ($\frac{N}{mm^2}$)
Sample 1	2,00	0,470	0,999	2,140	32,386
Sample 2	2,00	1,021	1,113	1,426	41,170
Sample 3	2,00	1,565	1,013	1,067	67,605

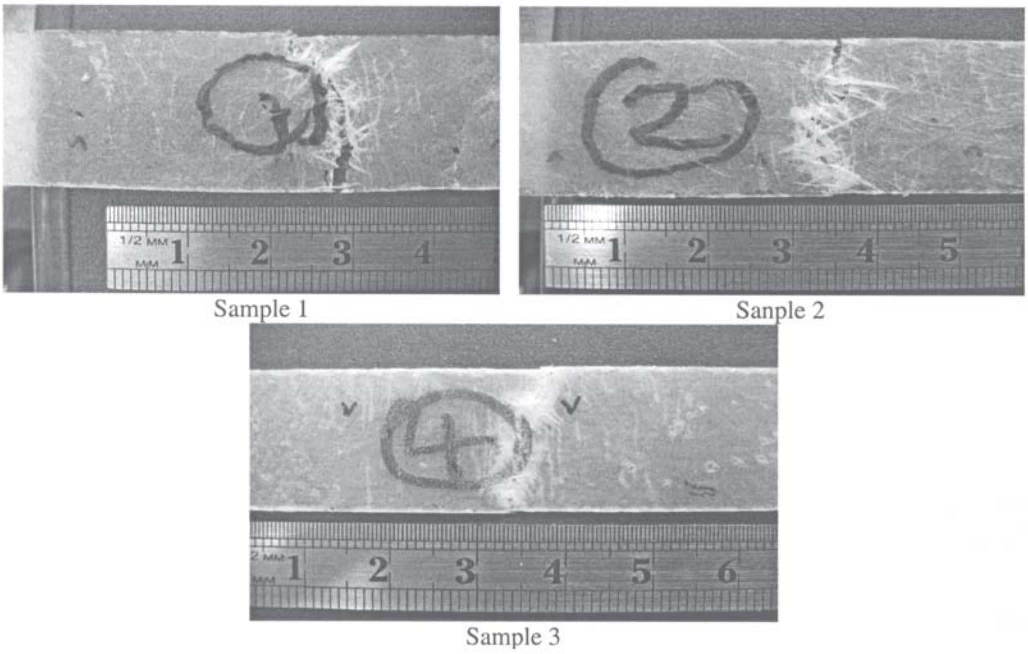

Fig. 3. The samples which was tested to traction

Sample 1

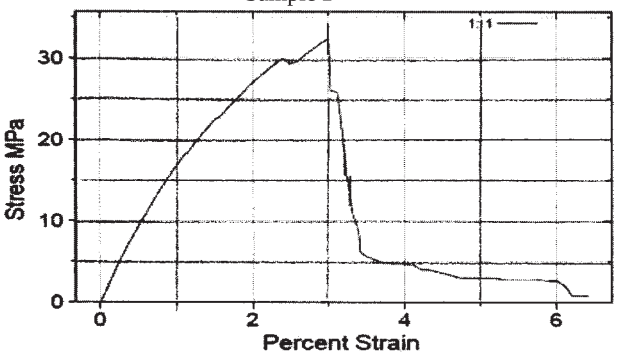

Fig. 4. Characteristic curve Sample 1

Sample 2

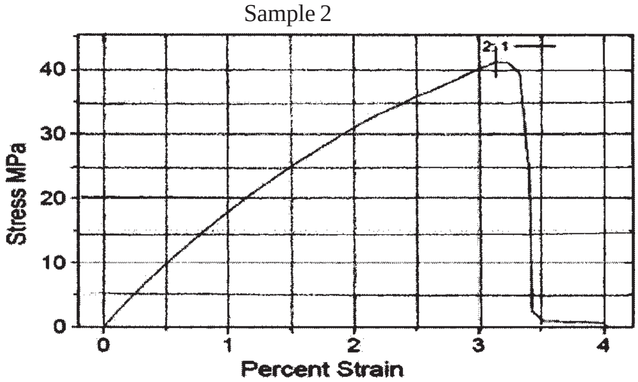

Fig. 5. Characteristic curve Sample 2

$$E = \frac{F \cdot l}{A \cdot \Delta l}, \qquad (11)$$

in which F is the traction load; l is the length of the test board subjected to stretching (the distance between the grips of the testing machine); A is the sectional area; Δl is the elongation of the specimen.

In table 3 it is presented a comparison between experimental results (for the failure strength and the elastic modulus) and theoretical ones, calculated with the relations (1) and (8) respectively.

One observes that the increase in the traction force and deformation coincides with a decrease of the elastic modulus under the values presented in table 3. The composite material thus registers a nonlinear behaviour due to the deformations which appear in the matrix, and to the fact that the fibres do not take over the stress evenly.

Sample 3

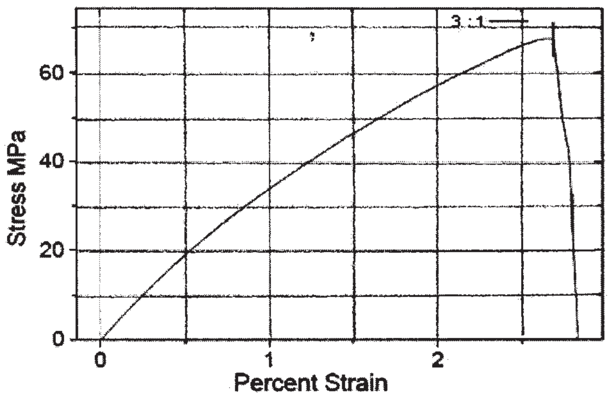

Fig. 6. Characteristic curve Sample 3

Conclusions

The analysis of the sections where the fracture occurs and the diagrams of stress-strain curves indicate that failures were sudden, without the existence of a flow area. A small difference appears in the case of sample 1 where the fibres did not break simultaneously, some fibres taking up the stress after the test board had given in. A similar phenomenon, but to a smaller extent, occurs in sample 2, whereas in the case of sample 3 all fibres can be said to have given in simultaneously. We may conclude that an increase in the volumetric proportion of fibres leads to a more homogeneous distribution of tensions which are evenly taken over by fibres. If the volumetric proportion of fibres is small, there is a greater heterogeneity and therefore areas of the composite material give in, in different manners.

The increase of the fibres volume fraction also leads to an increase of the failure strength as well as of the elongation to fracture and elastic modulus.

In case of composite materials with a random distribution of the reinforcement, the failure strength and the elastic modulus have lower values than in the case of unidirectional composites. This fact can be explained considering the smaller amount of fibres that take over the stress with respect to the total number of fibres disposed in the

Table 3

Order number	Volume fraction of the reinforcement	Experimental results		Theoretical results	
		Failure strength $(\frac{N}{mm^2})$	Elastic modulus (MPa)	Failure strength $(\frac{N}{mm^2})$	Elastic modulus (MPa)
Sample 1	0,1	32,38	1976	33,75	2044
Sample 2	0,15	41,17	2225	40,5	2304
Sample 3	0,3	67,61	3714	67,25	3678

composite. Moreover, the percentage of woven fibres is higher than in the case of composites with the reinforcement oriented on a given direction.

Since the composite plates with random distribution of reinforcement are assimilated as having isotropy in the plate plane, it results that they will have properties closer to matrix properties. This can be explained by the fact that, since the fibres modify their direction from one point to the other, they present a high degree of anisotropy.

References

1. HALPIN, J.C., ASHTON, J.E., PETIT, P.H., Primer on composite analysis, Technomic Publ. Co., Westport, 1969
2. HASKIN, Z., The elastic model of heterogeneous materials, ASME J. Appl. Mech., 29, 1962, p. 143
3. HILL, R., A self-consistent mechanics of composite materials, Mech. Phys. Solids, 13, 1965, p. 213
4. CHRISTENSEN, R., ZYWICS, E., Dimensional constitutive theory for liber composite laminated media, J. Appl., Mech., 57 , nr. 4, 1990, p. 948
5.CEDERBAUM, J., On the parametric instability of laminated plates modeled within a high order shear deformation theory, Acta Mechanica, 1992, 91, p. 179
6. WADDOUPS, M.E., Characterisation and design of composite materials, Composite Materials Workshops, (ed. S.W. Tsai, J.C. Halpin, N.Y. Pagano), Technomic Publ. Co., Standford, C.T., 1968, p. 254
7. PETIT, P.H., WADDOUPS, M.E., A method of predicting the nonlinear behavior of laminated composites, Compos. Mater., 3, 1973, p. 257
8. HAHN, H.T., Nonlinear behavior of laminated composites, Jour. Compos. Mater., 7, 1973, p. 257
9. JONES, R.M., MORGAN, H.S., Analysis of nonlinear stress-strain behavior of fiber-reinforced composite materials, AIAA Journal, 15, 1977, p. 1669
10.CIUCA, I., BOLCU, D., STĂNESCU, M.M., MARIN, GH., IONESCU, S.D., Mat.. Plast., **45**, nr. 3, 2008, p. 279
11.CIUCA, I., STĂNESCU, M.M., BOLCU, D., MOTOMANCEA, A., BUTU, M., Mat. Plast., **45**, nr. 4, 2008, p. 351
12.KACHANOV, L.M., Time of the rupture process creep condition, Izv. Akad. Nauk SSR, Otd. Tekh. Nauk, 8, 1958, p. 26
13.LADEVEZE, P., Sur une theorie de l'endommagement anisotrope, Rapport Interne No. 34, Laboratoire de Mecanique et Technologie, Cachan, France, 1983
14.LADEVEZE, P., Sur la mecanique de l'endommagement des composites, Compte-Rendus des JNC 5, Pluralis, Paris, 1986, p. 667
15.LADEVEZE, P., ALLIX, O., DAUDEVILLE, L., Mesomodeling of damage for laminate composites: application to delamination, Inelastic Deformation of Composite Materials, Springer-Verlag, New York, 1990, p. 607
16.ALLIX, O., LADEVEZE, P., Interlaminar interface modelling for the prediction of delamination, Comp. Struct., 22, 1992, p. 235
17.LADEVEZE, P., A damage computational method for composite structures, Comput. Struct., **44**, p. 79
18.ALLIX, O., BAHLOULI, N., LADEVEZE, P., PERRET, L., Damage modelling of composite laminates at various temperatures, in damage mechanics in composites, ASME, AMD, American Society of Mechanical Engineers, New York, 85, 1994, p. 21
19.HERAKOVICH, C., Mechanics of fibrous composites, John Wiley & Sons, New York, 1998
20.GAY, D., Materiaux composites, Edition Hermès, Paris, 1989

Manuscript received: 25.02.2008