The Experimental Validation of the Analytical Model, Used to Study the Dynamic Behaviour of a Multilayer Composite Structure

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In this paper is presented an analytical model, which makes the similitude between the DTMM theory of composite multilayer bars and the FSDT theory used in the study of composite bars and plates in a dynamic regime. The authors proposed a numerical application of the model, for two different bars made from glass-epoxy reinforced composite. The experimental validation of theoretical results was made on a composite bar as a component of a quadrilateral mechanism, whereon the leading element describes a rotation motion with the constant angular speed.

Keywords: composite materials, vibrations, movement in a plane

The dynamic studies on composite plates and bars take into consideration the following aspects:

- the statistical description of the load and proprieties of the composite structure;
- the determination of the dynamic response of the structure;
- the calculus of the structure reliability coefficient which defines the probability that the structure response will not exceed a certain level.

There can be mentioned the probabilistic studies [1-3], where the authors determined the elastic characteristics of composite materials.

The composite plates and bars could be analyzed using a lot of theories that differ mostly by the inclusions or neglecting the effects of angular deformation and rotational inertia respectively. The fundamentals of the classical lamination theory [4] are based on the hypothesis that a straight-line, normal on the median surface before deformation, remains straight and normal on the median surface also during deformation. For laminates with a ratio between the elasticity modulus E and shear modulus G reaching values of 25-40, be can proved that this theory overestimate the natural frequencies of the structure.

Another theory (First – order Shear Deformation Theory – FSDT) has been developed [5] and later modified [6]. This theory relies on a linear distribution of the shear stresses and requests a correction factor similar with the one from isotropic plates. This theory states that a straight line normal to median plane before deformation remains straight without keeping the normality on the median surface during deformation.

Exact theories rely on a non-linear distribution of shear stresses along the thickness of the plate or bar. The inclusion of high order terms implies the inclusion of supplementary unknowns. Moreover, when fulfilling both the distribution of shear stresses in thickness is parabolic and if the limit conditions on external surfaces, it is not necessary a correction factor. Based on this fact, it was developed a theory [7] (High – order Shear Deformation Theory – HSDT) where it is assumed that stresses and strains normal to the median plane are null. Another theory in which are considered also the stresses normal to the median plane has also been developed [8, 9] by removing a series of contradictions appearing in previous theories by accepting non linear factors of shear stresses in thickness; also, they did not neglect a part of the normal stresses obtained by the loading of the composite structure.

The case of elastic laminated plates loaded with periodical forces reaching Mathieu type equations was considered [10-12]. The dynamic stability of composite materials with visco-elastic behavior using Liapunov functions method was investigates [13], obtaining a numerical procedure that allows the determination of structure stability for a set of parameters. For composite materials like boron – epoxy resin or graphite – epoxy resin a non-linear behavior appears due to material matrix that affects mainly the shear modulus [14].

Similar results have been obtained in [15, 16].

The studies of the composite materials dynamics reserved a special place for sandwich bars made from several overlapped layers with similar thickness. Most studies refer to three layer sandwich bars, the middle layer having visco-elastic behavior and the inferior and superior layers having extra elastic and resilience proprieties. Other authors having similar studies on the behavior of these materials suggested the following:

- there is a continuity of displacements and stresses between layers;
- there is no deformation along the thickness of the bar;
- the transversal inertial forces are dominant, neglecting longitudinal inertia and rotational inertia of the bar section;
- the external layers have elastic behavior and are subjected to pure bending and the core has elastic or visco elastic behaviour taking over shear stresses;
- the core is not subjected to normal stresses.

Based on these hypotheses have been developed models considered to be the fundamentals of DTMM theory [17-19]. Starting from this theory, it was adapted a variation approach, obtained equations for sandwich plates taking

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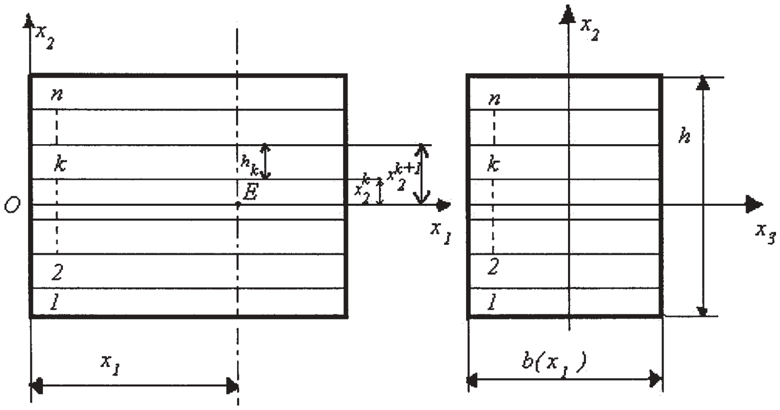

Fig. 1. Multilayer composite

also into consideration different angular deformations for the layers and managing to estimate the stresses between the layers [20].

Hypotheses of deformation and mathematical model

A particular composite with large applicability in practice is the multilayer composite. It consists of several layers of unidirectional composite having different compositions and orientations of the reinforcement, with different mass and elastic properties (fig. 1).

One of the assumptions is that the section of the bar is rectangular with thickness h=constant and width b = $b(x_1)$. We note by x_2^k the coordinate on Ox_2 axis of the inferior surface of the layer k and with h_k the thickness of this layer. We note by E_k and G_k the global elastic constants (longitudinal elastic modulus and shear modulus) and ρ_k the specific mass for the layer k. In addition, the layers can have many constituents, the overall elastic proprieties of the layer depending on the elastic proprieties of the constituents, on their volume fraction and geometrical distribution as well as on their adherence.

The position of the elastic center of the section can be determined from the condition:

$$\iint_{(S)} E \cdot X_2 \cdot dS = 0, \qquad (1)$$

from which results the height $x_2^{(1)}$ of the inferior surface of the bar referring to the elastic center

$$x_2^{(1)} = -\frac{2\sum_{k=1}^{n} E_k h_k \left(\sum_{i=1}^{k-1} h_i\right) + \sum_{k=1}^{n} E_k h_k^2}{2\sum_{k=1}^{n} E_k h_k}. \qquad (2)$$

The main characteristics of the bar's section will be:

$$\langle \rho A \rangle = \iint_{(S)} \rho dS, \qquad (3)$$

$$\langle EA \rangle = \iint_{(S)} E dS = b(x_1)\sum_{k=1}^{n} E_k h_k, \qquad (4)$$

$$\langle GA \rangle = \iint_{(S)} G dS = b(x_1)\sum_{k=1}^{n} G_k h_k, \qquad (5)$$

$$\langle EI \rangle = \iint_{(S)} E (x_2)^2 dS = b(x_1)\sum_{k=1}^{n} E_k \frac{\left(x_2^{k+1}\right)^3 - \left(x_2^k\right)^3}{3}, \qquad (6)$$

$$\langle \rho I \rangle = \iint_{(S)} \rho \left(x_2^2\right)^2 dS = b(x_1)\sum_{k=1}^{n} \rho_k \frac{\left(x_2^{k+1}\right)^3 - \left(x_2^k\right)^3}{3}. \qquad (7)$$

We notice that the first two characteristics $\langle EA \rangle$ and $\langle GA \rangle$ depend only on the elastic properties of the layers and their thickness, $\langle EI \rangle$ depending also on the way layers are arranged in the section. This gives the possibility that using the same layers, by changing their order, to obtain bars with different global properties, including those regarding the resilience and the dynamic behaviour having in this way the possibility of their optimization with respect to the technical demands.

We consider that the bar, subjected to analysis, has a motion in a plane parallel with the plane of symmetry of the bar Ox_1x_2.

The bar has longitudinal and transversal vibrations from the movement plane, neglecting the torsion and transversal vibrations normal on movement plane.

The following hypotheses on deformability are taken into account:

- any section of the bar, plane before deformation, moves and the sections of the component layers remain plane during deformation;
- the section is not rotating around axis Ox_1 and Ox_2;
- there are no discontinuities of the displacements on the separation surfaces between adjacent layers, thus presuming a perfect adherence between layers;

These hypotheses are specific to the jag model [20]. With these conditions, the field of displacements in k layer's section will have the form:

$$\begin{aligned} w_1^k(x_1,x_2,x_3,t) &= u_1^k(x_1,t) + \\ &+ \left(x_2 - x_2^k\right)\left(\Phi_k - u_{2,1}\right), \\ w_2^k(x_1,x_2,x_3,t) &= u_2(x_1,t),\ w_3^k(x_1,x_2,x_3,t) = 0, \end{aligned} \qquad (8)$$

where:

- u_1^k is the longitudinal displacement of k layer's inferior surface,
- Φ_k - the supplemental rotation of k layer's section due to this layer's deformation.

We know ([20]):

$$G_1\Phi_1 = G_k\Phi_k, (\forall) k \in \{1, 2, ..., n\}. \qquad (9)$$

This relation suggests that the shear stress is constant along the bar section.

Using the continuity conditions for the displacements on the separation surface between layers, could be obtained the longitudinal displacement of the points from the layer k with the form:

$$w_1^k = u_1^1 - \left(x_2 - x_2^{'}\right) u_{2.1} + + \left(x_2 \frac{G_1}{G_k} + G_1 B_k \right) \Phi_1 \quad (10)$$

where,

$$B_k = \sum_{i=1}^{k-1} h_i \left(\frac{1}{G_i} - \frac{1}{G_k} \right) - \frac{x_2^1}{G_k}. \quad (11)$$

We introduce:

-longitudinal displacement of the section:

$$u_1 = u_1^1 + x_2^1 \cdot u_{2,1} + G_1 \Phi_1 B \quad (12)$$

where,

$$B = \frac{1}{\sum_{k=1}^{n} E_k h_k} \cdot \sum_{k=1}^{n} [E_k B_k h_k + + \frac{E_k}{G_k} \cdot \frac{\left(x_2^{k+1}\right)^2 - \left(x_2^k\right)^2}{2}]; \quad (13)$$

-the rotation of the section:

$$\theta_3 = u_{2,1} - G_1 C \Phi_1 \quad (14)$$

where

$$C = \frac{b(x_1)}{\langle EI \rangle} \sum_{k=1}^{n} [\frac{E_k}{G_k} \cdot \frac{\left(x_2^{k+1}\right)^3 - \left(x_2^k\right)^3}{3} + + E_k B_k \cdot \frac{\left(x_2^{k+1}\right)^2 - \left(x_2^k\right)^2}{2}]. \quad (15)$$

Using the displacement u_1 and the rotation θ_3 of the section calculated this way we can write the displacement of a point from the layer k as follows:

$$w_1^k = u_1 - x_2 \theta_3 - -G_1 \Phi_1 \left[B_k - B + x_2 \left(\frac{1}{G_k} + C \right) \right]. \quad (16)$$

The introduction of longitudinal displacement (12) and rotation of the section (14) connects the jag model, DTMM and FSDT theories.

In [21] is presented the mathematical model for the vibrations of a composite bar moving in a plane. In case of thin composite bars moving in a plane one could accept $\theta = u_{2,1}$ and the vibrations equations take the next form:

$$\begin{aligned} &\langle \rho A \rangle \ddot{u}_1 - 2\langle \rho A \rangle \omega \dot{u}_2 - \langle \rho A \rangle \omega^2 u_1 - \\ &-\langle \rho A \rangle \varepsilon u_2 - \langle EA \rangle u_{1,11} = \\ &= p_1 - \langle \rho A \rangle \left(a_{01} - \omega^2 x_1 \right), \end{aligned} \quad (17)$$

$$\begin{aligned} &\langle \rho A \rangle \ddot{u}_2 - \langle \rho I \rangle \ddot{u}_{2,11} + 2\langle \rho A \rangle \omega \dot{u}_1 + \\ &+\langle \rho A \rangle \varepsilon u_1 - \langle \rho A \rangle \omega^2 u_2 + \\ &+\langle \rho I \rangle \omega^2 u_{2,11} + \langle EI \rangle u_{2,1111} = \\ &= p_2 - m_{3,1} - \langle \rho A \rangle \left(a_{02} - \varepsilon x_1 \right), \end{aligned} \quad (18)$$

where:

- ω is the angular velocity of the bar,
- ε is the angular acceleration of the bar,
- a_{01}, a_{02} are the acceleration components of the Cartesian coordinate system's origin of the bar,
- p_1, p_2 are external forces distributed on the length of the bar,
- m_3 are the external moments distributed on the length of the bar.

In [22] one could find a study of the operator associated with equation (18), in a general form, from the spectral point of view.

Next, it will be studied the case of a composite bar in translation movement, with the boundary conditions (for $x_1=0$) and $x_1=L$:

-for longitudinal vibrations

$$u_1(0;t) = u_1(L;t) = 0; \quad (19)$$

-for transversal vibrations

$$\begin{aligned} &u_2(0;t) = u_2(L;t) = 0, \\ &u_{2,11}(0;t) = u_2(L,t) = 0. \end{aligned} \quad (20)$$

We will consider null initial conditions both for transversal vibrations as well as for longitudinal vibrations.

In these conditions the deformations of the translating composite bar are:

-the longitudinal deformation

$$u_1(x_1,t) = \frac{2}{L} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{\omega_n} \cdot \cdot \left(\int_0^t \sin\left(\omega_n (t-\tau)\right) a_{01}(\tau) d\tau \right) \cdot \sin \frac{n\pi x_1}{L}, \quad (21)$$

where,

$$\omega_n = \frac{n\pi}{L} \sqrt{\frac{\langle EA \rangle}{\langle \rho A \rangle}} (22) \quad (22)$$

is the set of its own pulsations for the longitudinal vibrations,

-transversal deformation:

$$u_2(x_1,t) = \frac{2}{L} \sum_{n=1}^{\infty} \frac{\left((-1)^n - 1\right) L}{n\pi \Omega_n \left(1 + \frac{n^2 \pi^2}{L^2} \frac{\langle \rho I \rangle}{\langle \rho A \rangle} \right)} \cdot \cdot \left(\int_0^t \sin\left(\Omega_n (t-\tau)\right) a_{02}(\tau) d\tau \right) \cdot \sin \frac{n\pi x_1}{L}, \quad (23)$$

where,

$$\Omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{\langle EI \rangle}{\langle \rho A \rangle + \langle \rho I \rangle \frac{n^2 \pi^2}{L^2}}} \quad (24)$$

is the set of own pulsations for the transversal vibrations.

Numerical application and experimental verifications

The numerical calculus and experimental validations have been done for composite bars having an epoxy resin matrix reinforced with fiberglass stripes, composing a cloth arranged in layers. The figure 2 shows a section photo where is easy to be seen how the glass fibers fit in the matrix, the volume fraction of the fibers being $V_f = 0.6$.

The main elastic characteristics for the epoxy resin are:

- elastic modulus (Young)

E_m = 4500 MPa;

- transversal elasticity modulus (shear)

G_m = 41600 MPa;

- specific mass

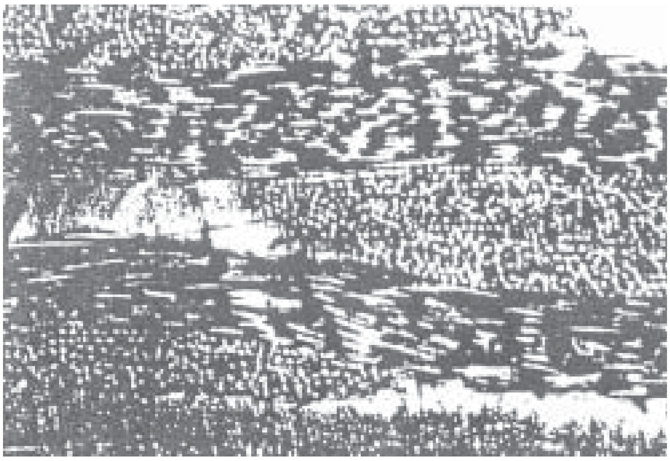

Fig. 2. The arrangement of the glass fibres in the matrix

ρ_m = 1200 kg/m^3;
- transversal contraction coefficient (Poisson)
γ_m = 0.4
-compressibility coefficient
K_m = 7500 MPa.
The main elastic characteristics for the glass fibers are:
-elastic modulus:
E_f = 73000 MPa;
-shear modulus:
G_f = 30000 MPa;
- specific mass:
ρ_f = 2500 kg/m^3;
-Poisson coefficient
γ_f = 0.25 ;
-compressibility coefficient
K_f = 4900 MPa.

In table 1 are presented the results for two different bars from the same material with different transversal dimensions.

The own pulsations for longitudinal vibrations have the same values for both bars. The first six values are:

$$\omega_1 = 33280\ s^{-1};\ \omega_2 = 66560\ s^{-1};$$
$$\omega_3 = 99840\ s^{-1};\ \omega_4 = 133120\ s^{-1};$$
$$\omega_5 = 166400\ s^{-1};\ \omega_6 = 199680\ s^{-1}.$$

Table 1

Characteristic	Bar 1	Bar 2
$\langle EI\rangle\left[\left(N/m^2\right)\cdot m^4\right]$	5,63	77,76
$\langle \rho A\rangle\left[\left(Kg/m^3\right)\cdot m^2\right]$	$1188\cdot 10^{-4}$	$2851{,}2\cdot 10^{-4}$
$\langle EA\rangle\left[\left(N/m^2\right)\cdot m^2\right]$	$270\cdot 10^{4}$	$648\cdot 10^{4}$
$\langle \rho I\rangle\left[\left(Kg/m^3\right)\cdot m^4\right]$	$247{,}5\cdot 10^{-9}$	$3421{,}4\cdot 10^{-9}$
$\langle GA\rangle\left[\left(N/m^2\right)\cdot m^2\right]$	$318\cdot 10^{3}$	$736{,}2\cdot 10^{3}$

The own pulsations for transversal vibrations are different for each bar.

For the bar 1 the first six values are:

$$\Omega_1 = 335{,}4\ s^{-1};\ \Omega_2 = 1341{,}2\ s^{-1};$$
$$\Omega_3 = 3017\ s^{-1};\ \Omega_4 = 5361{,}6\ s^{-1};$$
$$\Omega_5 = 8373{,}7\ s^{-1};\ \Omega_6 = 12051{,}4\ s^{-1}.$$

For the bar 2 the first six values are:

$$\Omega_1 = 804{,}6\ s^{-1};\ \Omega_2 = 3215{,}8\ s^{-1};$$
$$\Omega_3 = 7225\ s^{-1};\ \Omega_4 = 12818{,}5\ s^{-1};$$
$$\Omega_5 = 19976\ s^{-1};\ \Omega_6 = 28675\ s^{-1}.$$

We notice that the bar own pulsations in case of longitudinal vibrations have very high values so practically having a resonance for this kind of vibrations did not represent a danger. It is not the same case for the transversal vibrations where the first own pulsations are in the interval of those used in technology. For the case of the own pulsations of transversal vibrations we notice that the term with the coefficient $\langle\rho I\rangle$ has a significant influence only for high level pulsations and can be neglected in many technological applications.

We study the case of a composite bar as a part of a quadrilateral mechanism structure where the leading element rotates with constant angular velocity p = 25π s^{-1}.

The following values have been obtained:

Longitudinal displacement:

$$u_1(x_1,t) = \sum_{n=1}^{\infty} \frac{149{,}2}{n\left[(66960n)^2 - 625\pi^2\right]}\cdot \left(\cos 25\pi t - \cos 66560nt\right)\cdot \sin\frac{n\pi x_1}{0{,}225}; \quad (25)$$

the maximal values of the displacement for the first two harmonics of the development are:

for $n=1$ $\quad u_{1\max} = 3{,}32\cdot 10^{-8}\ m;$

for $n=2$ $\quad u_{1\max} = 4{,}16\cdot 10^{-9}\ m;.$

One can notice that the longitudinal deformations can be practically neglected.

Transversal displacement:

- for bar 1

$$u_2(x_1,t) = \sum_{n=1}^{\infty} \frac{149{,}2\cdot\left(1-(-1)^n\right)}{n\Omega_n\left(1+n^2\cdot 1{,}015\cdot 10^{-4}\right)}\cdot \left[\frac{\Omega_n \sin(25\pi t) - 25\pi \sin(\Omega_n t)}{\Omega_n^2 - 625\pi^2}\right]\cdot \sin\frac{n\pi x_1}{L}; \quad (26)$$

the maximal values of the displacement for the first two harmonics of the development are:

- for $n=1$ $\quad u_{2\max} = 0{,}002825\ m;$
- for $n=3$ $\quad u_{2\max} = 3{,}53\cdot 10^{-5}\ m;$

- for bar 2

$$u_2(x_1,t) = \sum_{n=1}^{\infty} \frac{149{,}2\cdot\left(1-(-1)^n\right)}{n\Omega_n\left(1+n^2\cdot 5{,}85\cdot 10^{-4}\right)}\cdot \left[\frac{\Omega_n \sin(25\pi t) - 25\pi \sin(\Omega_n t)}{\Omega_n^2 - 625\pi^2}\right]\cdot \sin\frac{n\pi x_1}{L}; \quad (27)$$

the maximal values of the displacement for the first two harmonics of the development are:

- for $n=1$ $\quad u_{2\max} = 4{,}68\cdot 10^{-4}\ m;$
- for $n=3$ $\quad u_{2\max} = 5{,}21\cdot 10^{-5}\ m;.$

It ca be noticed that the second harmonic's amplitude is about nine times smaller then the first harmonic's amplitude and in this case we can approximate the

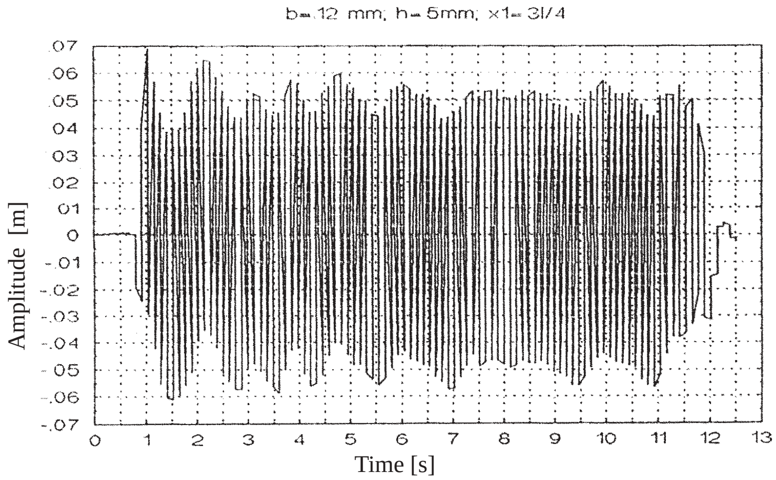

Fig. 3. The variation of bar 2 deformation for its middle section

amplitude of the bar's vibration with the amplitude of the first harmonics.

Experimental determinations with the vibration analyzer BK 2515 have been done, obtaining the measurements in the time mode within the range 0 - 12.5 s. We processed the mechanical signals for displacement with the 4391 piezo-electric transducer having the sensitivity 1±0.02 pC/ms.

For data processing, we used BK 7616 software and the functions of IEEE – 488 interfaces.

In figure 3 is presented the variation of bar 2 deformations for the middle section.

The study of vibrations from structures built from composite materials has immediate applications in aeronautics due to the fact that these structures insure a very good report between the mechanical resilience and the quantity of material used. Thus, favorable consequences on cost efficiency and reliability of airplanes and rockets are obtained.

In naval domain, these structures could be used to build light boats, obtaining high rigidity for a low total mass, creating the conditions for raising speed, payload as well as reduction of fuel consumption.

The composite materials are often subjected to vibrations in parts like clutch disks, body parts, auxiliary connecting rods and others.

In the medical field, we find the composite materials used for prosthetics and implants because they are chemically stable and do not introduce negative effects, the human body easily tolerating them.

In sports we can use them to make tennis racket, skies, bicycle frames, protection helmets.

In all these fields, using composite materials with appropriate dynamic response according to certain specific conditions represents a way for rational and integral use of materials.

Conclusions

After analyzing the theoretical and experimental results, the following conclusions are presented below:

-the bar own pulsations for its longitudinal vibrations do not depend neither on the size of the section nor on the composing layers' geometrical distribution;

-the bar own pulsations for its transversal vibrations depend on the way the layers are arranged in the section and can be modified by changing only the layers order;

-the bar own pulsations for its longitudinal vibrations have very high values that cannot be found in most applications so the danger of having a resonance for this type of vibrations is very low; it is not the same case on the transversal vibrations for which the first own pulsations are under $1000\ s^{-1}$;

-the term containing $\langle \rho I \rangle$ characteristic has an influence only for high values of "n" so in practical applications can be neglected;

-the compared study of the transversal vibrations for the two bars shows that the amplitude decreases when the values of geometric-elastic characteristics of the section are raising; for the bar own pulsations of the transversal vibrations a reverse phenomenon appears, meaning that amplitude raises with the raise of the bar section characteristic values.

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Manuscript received: 10.03.2008