Nonlinear Dynamics of Steel/Plastic Gears of Servomechanism Structures

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The paper presents the main aspects of dynamic modelling of steel/plastic geared systems with high gear ratios in relation to the nonlinear parameters, such as time-varying mesh stiffness and load sharing alternation. Firstly, the specific aspects of geometrical design criteria for helical gear with small number of pinion teeth are presented in order to prevent the negative phenomena of the involute tooth generation and gear engagement. Then, the dynamic analysis is conducted to investigate the shared dynamic loads and dynamic transmission error of steel/plastic gear pairs along the line of action. The effects of the time-varying mesh stiffness are incorporated in the nonlinear dynamic model for numerical analysis. An improved model of the mesh stiffness is developed by including specific meshing characteristics of helical gear pairs. The variation of the dynamic transmission error is simulated and examined in relation to the teeth number, addendum modification coefficients and input torques. A computational algorithm is developed for instantaneous dynamic contact analysis.

Keywords: plastic gears, high gear ratio, mesh stiffness, dynamic transmission error

Plastic gears have many advantages comparatively with steel gears, such as reduced weight, self-lubrication, and lower manufacturing costs. However, the applications of plastic gears are limited due to a low load-carrying capacity and sensibility to increased temperature conditions. Previous studies on plastic gears investigated various aspects, including tooth wear [1-4], power transmission [5], tooth strength [6], static transmission errors [7,8], and acoustic emission [9].

Gear transmissions are used in the structure of specific servomechanisms, such as robotic applications, with small size actuators to achieve high driving torques and reduced angular speed at the end-effector. This demand can be achieved by using gear transmissions with high gear ratio. The gear pairs with a driver gear (pinion) having small number of teeth can realize transmissions with gear ratio $u > 12$ in a single stage. The gear transmissions in which the pinion with small number of teeth is made from steel and the gear is made from plastic have the advantage of a high gear ratio, reduced weight and compact structures.

There have been several researches conducted on the specific aspects of the steel gears with small number of teeth, such as specific geometric relationships [10-12], load capacity [13,14], and manufacturing modeling [15].

The high performance of geared servomechanisms is strongly affected by the dynamic characteristics of their mechanical systems. An accurate dynamic modeling of these geared systems becomes important in order to achieve the increasing demand of high accuracy for predicted dynamic characteristics.

Studies on the dynamics of helical gears and small number of pinion teeth and the combination of material as steel/plastic are not published. The dynamic modeling of these gear pairs becomes important in order to extend their applications for geared servomechanisms. The dynamic transmission error is considered as a significant parameter for the design and control of these devices [16-17].

This paper presents a representative and suitable method for calculating the dynamic transmission error of steel/plastic gear transmissions with small number of pinion teeth. Firstly, the main specific aspects of geometrical design criteria for helical gear with small number of pinion are presented. Then, the dynamic analysis is conducted to investigate the shared dynamic loads of steel/plastic gear pairs along the line of action.

The engagement accuracy under dynamic conditions is a result of the instantaneous contact conditions between active tooth profiles. In the present paper, the dynamic transmission error of gear systems is directly related to the deviation of the gear tooth profiles in the form of the elastic deformation and shared dynamic loads of gear tooth.

The position accuracy in a motion control system is affected by vibration due to the nonlinear effect of mesh stiffness. The time-varying mesh stiffness represents the main cause of undesired vibrations in the case of gear transmissions. An analysis of the influence of the mesh stiffness of the steel/plastic gears on the dynamic motion characteristics is presented in the paper in relation to the number of teeth and addendum modification coefficients. In order to obtain reliable data for the prediction of gear dynamic behaviour, the dynamic model accounts a non-linear time varying mesh stiffness. The time-variation of contact length is included in this model. The calculus procedure of the mesh stiffness discussed in the paper is for involute helical gears, which are generated by using the basic rack.

Geometrical Design Criteria

The geometrical criteria are used to prevent the negative phenomena of the generating and engagement processes of helical gears with small number of teeth. Thus, in the design stage, the following condition must be accomplished:

$$x_n \geq x_{n\,min} \qquad (1)$$

where $x_{n\,min}$ represents the minimum value of the addendum modification coefficient to avoid tooth undercutting [11] and can be expressed as:

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$$x_{n\min} = h^*_{oa} - \rho^*_{on}(1-A) - \frac{z\sin^2\alpha_t}{2\cos\beta} \quad (2a)$$

$$A = \sin\alpha_t[\sin^2\alpha_t + (\cos\alpha_t/\cos\beta)^2]^{0.5} \quad (2b)$$

where z represents the number of pinion teeth, h^*_{oa} is the reference tool addendum coefficient, ρ^*_{on} is the reference radius of the generating tool, α_t is the transverse pressure angle and β represents the helix angle. In order to maintain a reasonable value of the tooth thickness s_{an} on the outside circle it is recommended the following condition:

$$s_{an} \geq \delta m_n \quad (3)$$

where δ=0.25 for quenched and tempered steel gears and m_n represents the tooth module. For a helical gears with a small number of teeth, when eq. (1) is accomplished but eq. (3) is not carried out, it is necessary to reduce the whole addendum with a value $\Delta_a m_n$ to avoid tooth top sharpening. In such cases, the outside diameter is expressed as:

$$d_a = m_t z + 2m_n(h^*_{an} + x_n - \Delta_a) \quad (4)$$

and

$$F(x_n, \Delta_a) = s_{an} - \delta m_n \quad (5)$$

where m_n, m_t are the normal and transverse tooth module and s_{an} is the top land tooth thickness. In the design stage it is considered a value $x_n \geq x_{n\min}$ and the Δ_a coefficient is computed on basis of eq.(5) by using a numerical method.

The total contact ratio ε_γ must be always bigger than the unity, in order to preserve the continuity of engagement gears, where

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \quad (6)$$

where ε_α represents the transverse contact ratio and ε_β is the overlap contact ratio.

Dynamic Model

The characteristic of the helical gears is mainly involved in the inclination of the contact lines. Both the position and the length of the contact line at time t have a significant influence on the amount of the mesh stiffness of the gears. Therefore, it is necessary to establish these parameters with high accuracy.

Figure 8 shows some of the relevant features of the meshing plane of action for a pair of helical gears. The meshing starts at point A, passes through point E_t and finishes at point E_x. In the analysis it is useful to consider an equivalent line of action AE as shown in figure 1.

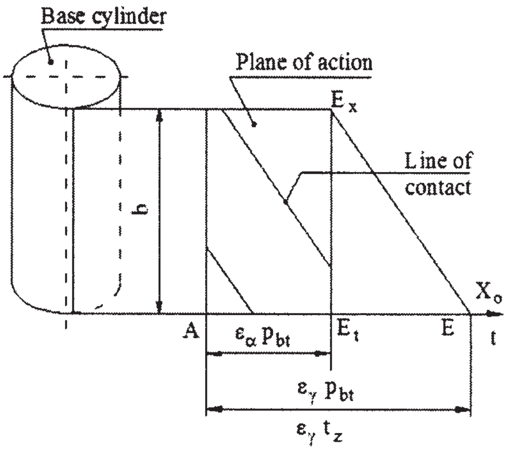

Fig. 1. The equivalent line of action of a helical gear

The position of the line of contact is indicated with one of the coordinate X_o or t of the equivalent line of action, where X_o shows the meshing position and t is the meshing time:

$$X_o = \varepsilon_o p_{bt}, \quad t = \varepsilon_o t_z \quad (7)$$

In the above equations, $\varepsilon_o = 0 \sim \varepsilon_\gamma$, and t_z is the meshing time period of passing a transverse base pitch p_{bt} of the helical gears.

The typical dynamic model for a gear pair in mesh is shown in figure 2. In this model, the teeth are considered as springs and the gear blanks as inertia masses. The gear mesh interface is represented by the time-varying mesh stiffness $k_i(t)$, the viscous damper c and the composite tooth profile error $\varepsilon_i(t)$. The motion transfer along the line of action between the two gears represents the dynamic transmission error (DTE) and can be expressed as $\delta = r_{b1}\theta_1 - r_{b2}\theta_2$. The differential equations of motion can be expressed as:

$$J_1\ddot{\theta}_1 + c\dot{\delta}(t)r_{b1} + \sum_{i=I}^{N} r_{b1}F_{di} = T_1 \quad (8)$$

$$J_2\ddot{\theta}_2 - c\dot{\delta}(t)r_{b2} - \sum_{i=I}^{N} r_{b2}F_{di} = T_2 \quad (9)$$

where θ_1, θ_2 are the rotation angle of the pinion and the driven gear, respectively, J_1 and J_2 are the mass moments of inertia of the gears, T_1 and T_2 denote the external torques applied on the gear system, and r_{b1}, r_{b2} are the base circle radii of the gears.

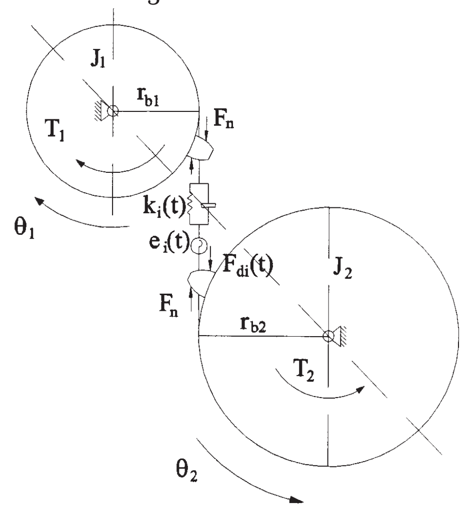

Fig. 2. Dynamic model of a helical gear pair

Eqs. (8) and (9) can be reduced into one equation of translation motion in the following form:

$$m_e\ddot{\delta}(t) + c\dot{\delta}(t) + \sum_{i=I}^{N} F_{di}(t) = F_n \quad (10)$$

where F_n is the external force. The dynamic normal load between two meshing gear teeth is expressed as:

$$F_{di} = k_i(t)(\delta(t) - e_i(t)) \quad (11)$$

In eq.(10) c is the damping coefficient calculated by

$$c = 2\xi\sqrt{m_e k_m} \quad (12)$$

where ξ represents the damping ratio factor, k_m is the average mesh stiffness of the gear pair and m_e represents the equivalent inertia mass

$$m_e = \frac{J_1 J_2}{J_1 r_{b1}^2 + J_2 r_{b2}^2} \quad (13)$$

Parameters of the dynamic model

Time varying contact length

The time-variation of the contact length is one of the most important parameter in the analysis of the cyclic mesh stiffness and the contact pressure of meshing tooth pairs. For helical gears, the contact length of a tooth pair is not a constant during the engagement cycle. The time-varying of the length of a contact line depends on the ratio between the transverse contact ratio ε_α and the overlap contact ratio ε_β according to the meshing phase and can be expressed as

$$l_c = \frac{\varepsilon_\alpha}{\varepsilon_\beta} \cdot \frac{b}{\cos \beta_b} \cdot c_x \quad (14)$$

where *b* is the face-width of a gear pair, β_b is the helix angle on the base cylinder, and the parameter $c_x = f(\varepsilon_\alpha, \varepsilon_\beta)$ is shown [11]. The total contact length L_c at time *t* is the sum of the contact lengths of each of the tooth pairs

$$L_c = \sum_{j=1}^{n} l_{cj} \quad (15)$$

where *n* is the total number of teeth pairs simultaneously in mesh.

Time – Varying Mesh Stiffness

The time-varying mesh stiffness represents one of the main causes of undesired vibrations of gear transmissions. Some reported studies [18,19] have assumed a constant mesh stiffness in the dynamic analysis of helical gears. Within [20] it is presented an approximate stiffness function of helical gears, but this model cannot be extended for helical gears with small number of teeth and high gear ratio. In order to calculate the dynamic transmission error, the estimation of the time-varying mesh stiffness is required.

A general model to calculate the cyclic meshing stiffness of spur gears is developed in the paper. The time-varying mesh stiffness is mainly caused by the following factors: (i) the variation of the single mesh stiffness along the line of action; (ii) the fluctuation of the total number of total pairs in contact during the engagement cycle. The effect of bending, shear and Hertzian contact deformation is taken into account in the analytical method to calculate the tooth deformation.

The gear tooth is modeled to be a nonuniform cantilever beam supported by a flexible fillet region and foundation [21] as shown in figure 3. The tooth mesh stiffness is defined in the normal direction to the contact surface. The deflection f_i of the tooth is defined as the displacement of the point of applied load in the direction of the load. The individual tooth stiffness k_j can be denoted as $k_j = W_n / f_j$, where W_n is the normal tooth load per unit length $W_n = F_n / l_{ci}$

The deflection f_i of a pair of meshing teeth is expressed as

$$f_j = \sum_{n=1}^{2} f_{zj} + f_H \quad (16)$$

where f_{zj} is the deflection due to bending, shear and axial deformation of the tooth corresponding to the tooth profile, and f_H is the local compliance of the Hertzian contact. The tooth deflections are analytically derived by using the method of the potential energy of deformation. The analytical modelling considers the real tooth profile and the geometrical parameters which are specific for helical gears with small number of pinion teeth. The tooth deflection f_z is analytically derived by using the method of the potential energy of deformation applied for steel gears, and can be expressed in the integral form as follows:

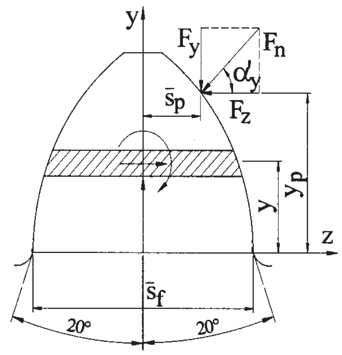

Fig.3. Gear tooth deflection model

$$f_z = \frac{F_n \cos^2 \alpha'_y}{E_e} \left(\int_0^{y_p} \frac{M'_i}{I_y} dy + \int_0^{y_p} \frac{2.4(1+\nu) + \tan^2 \alpha'_y}{A_y} dy \right) \quad (17)$$

where

$$M'_i = (y_p - y)\cos \alpha'_y - \bar{s}_f \cdot \sin \alpha'_y \quad (18)$$

$$E_e = 2\frac{E_1 E_2}{E_1 + E_2} \quad (19)$$

In eq.19, E_1 and E_{12} represent the effective modulus of elasticity of the pinion and driven gear, respectively. The tooth parameter $s_f = f(y)$ and integrands, I_y and A_y, are formulated in terms of variable with great complexity [21]. The equations of the tooth profile coordinates are established from the geometrical conditions by using the property of the involute profile and trochoidal curve corresponding to the fillet profile.

The mesh stiffness k_i corresponding to a contact line is calculated by using an iterative procedure. Thus, the contact line of a pair of teeth is divided into many equal intervals and k_i is computed as the integral value of the mesh stiffness k_j as it follows:

$$k_i = \int_0^{l_c} k_j \cdot dl \quad (20)$$

The length of each contact line changes during the meshing. This procedure leads to an average mesh stiffness corresponding to a contact line. In an alternative procedure developed in the paper, the tooth mesh stiffness k_j is computed in the middle of a contact line length and is corroborated with the length of the instantaneous contact line. It should be noted that this procedure significantly reduces the computational time. The numerical results obtained from the both procedures are very close.

The teeth pairs, when are in contact act like parallel springs. Therefore, the sum of individual mesh stiffness for all pairs in contact at time *t* represents the variable mesh stiffness $k_t(t)$ and can be written as:

$$k_t(t) = \sum_{i=1}^{n} k_i(t) \quad (21)$$

where *n* is the total number of teeth pairs in mesh in the same time. The average value of the mesh stiffness k_m can be expressed as:

Table 1
SPECIFICATIONS OF THE HELICAL GEAR PAIRS

Gear Pair	GP1	GP2	GP3	GP4
Number of pinion teeth, z_1	4	4	6	6
Number of pinion teeth, z_2	51	51	77	77
Tooth normal module, m_n, [mm]	1	1	1.5	1.5
Helix angle, β	20^o	20^o	20^o	20^o
Addendum modification coefficient x_{n1}	0.74*	0.85	0.6*	0.85
Addendum modification coefficient x_{n2}	0.05	-0.06	0.29	0.04
Tooth face-width, b [mm]	16	16	16	16
Transverse contact ratio, ε_α	0.89	0.82	1.08	0.94
Overlap ratio, ε_β	1.16	1.16	1.74	1.74
Total contact ratio, ε_γ	2.05	1.98	2.82	2.68
Material	steel/plastic	steel/plastic	steel/plastic	steel/plastic
Young's modulus, E_1 / E_2	207/3.54	207/3.54	207/3.54	207/3.54
Poisson's ratio, ν_1 / ν_2	0.3/0.35	0.3/0.35	0.3/0.35	0.3/0.35
* x_{n1min}				

$$k_m = \frac{1}{t_z} \int_0^{t_z} \sum_{i=1}^{n} k_i(t) dt \quad (22)$$

where $k_i(t)$ is the stiffness of the i -th meshing tooth pair.

Numerical Results

Specifications of the gear pairs which are selected in the analysis are shown in table 1. Additionally, the design parameters are chosen as: nominal torque, T_1 = 0.3 / 0.42 [Nm]; pinion speed, n_1 = 1800 rpm; damping ratio, ξ=0.16. In the simulation, steel was considered as the material for pinion and Nylon 66 for the driven gear. The elastic proprieties for driven gear material are E_2 = 3.54 [GPa] and ν = 0.35 [7].

The contact ratio of helical gear pairs can be influenced by parameters such as number of teeth and addendum modification coefficients. For the given centre distance and gear ratio results the sum of addendum modification coefficients as $x_{ns} = x_{n1} + x_{n2}$. The distribution of the sum x_{ns} between pinion and driven gear is influenced by the geometrical conditions, specific sliding and dynamic characteristics. The tribological analysis evinced that it is desirable to use an addendum modification coefficient x_{n1} a little larger than the limit of the tooth undercutting x_{n1min} in order to reduce the specific slidings and the tooth wear.

The amount of the addendum modification coefficients x_{n1} and x_{n2} allows to change the transverse contact ratio. The overlap contact ratio is dependent on the tooth face-width. The selected values of the addendum modification coefficients permit the obtaining of different values for the total contact ratio. In order to avoid the larger deflection of the tooth pair corresponding to the meshing zone of single contacting pair, the gear pairs GP1, GP3, GP4 have the total contact ratio in the range of 2.05 to 2.82.

The gear mesh stiffness is dependent on the total contact ratio. At the same time, by varying the total contact ratio, the dynamic normal load on the length of line of contact is not a constant. These aspects are specific ones in the analysis of helical gear pairs.

A computer program was developed for simulating the dynamic characteristics of specified gear pairs. It calculated instantaneously contact lengths, time-varying mesh stiffness, shared dynamic loads, and dynamic transmission error. The equations of motion are solved by the fourth-order Runge-Kutta method. In the following, the dynamic characteristics are calculated at the middle of the instantaneous contact line in order to consider the specific aspects of the engagement process of the helical gears.

Examples of the variation of the individual and total mesh stiffness from the starting to ending of the path of contact are shown in figure 4, where k_I, k_{II}, K_{III} represents the mesh stiffness for the tooth pairs *I, II, III* simultaneous in contact, and k_t represents the total mesh stiffness of the gear pair. These numerical results show the effect of the number of teeth on the variation of the amount and the shape of individual and total mesh stiffness variation during a mesh cycle. The amount of mesh stiffness is in good agreement with the ones estimated by previous researches [7,8].

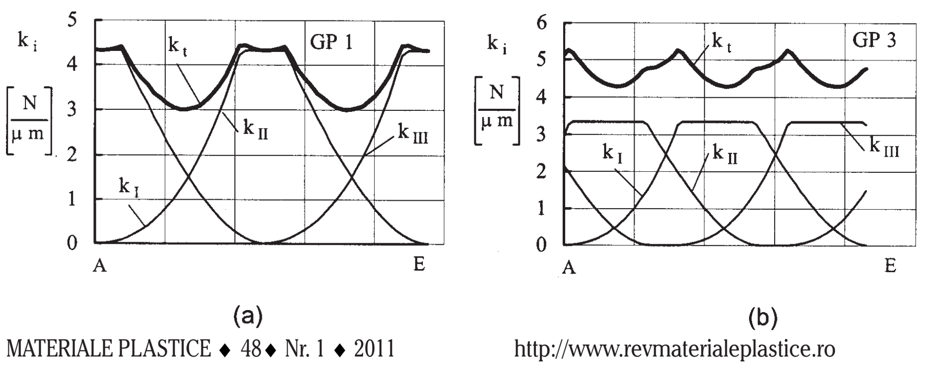

Fig. 4. Variation of the mesh stiffness along the line of action

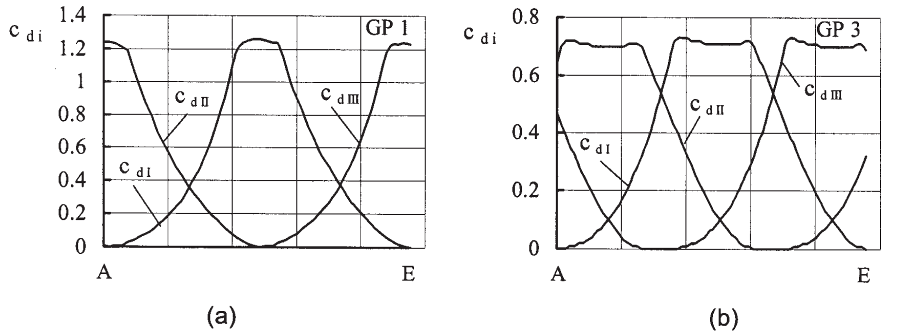

Fig. 5. Variations of dynamic factors

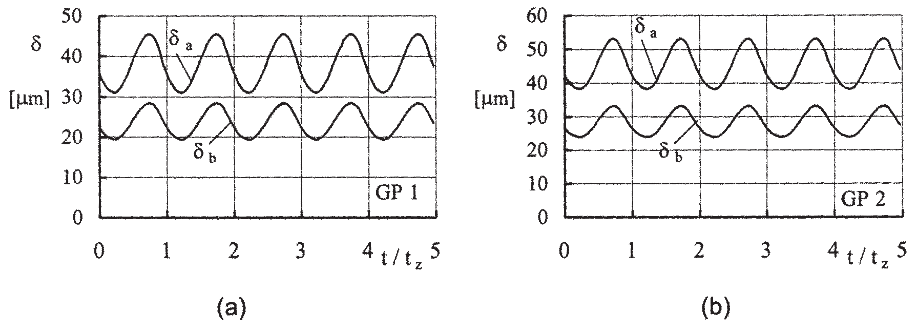

Fig. 6. Variations of the dynamic transmission errors

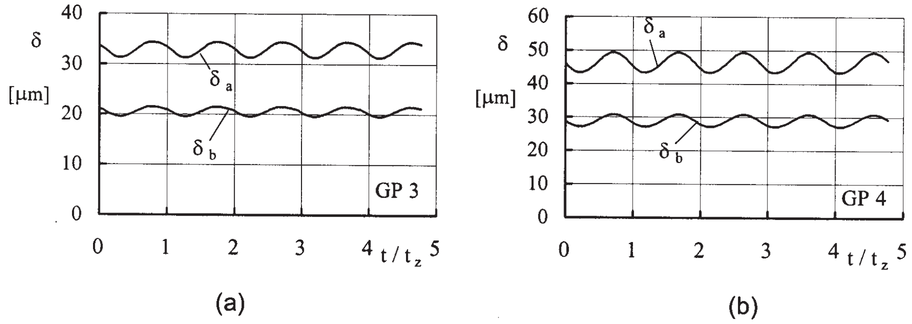

Fig. 7. Variations of the dynamic transmission errors

Shared dynamic loads

The meshing dynamics of the gear pairs can be expressed by the dynamic load factors c_{di}, where c_{di} correspond to shared dynamic loads on gear teeth. The dynamic factor is defined as the ratio of the dynamic load to the nominal static load transmitted by the gear pair. Examples of the variations of the dynamic factors c_{di} are shown in figure 5, where i = I, II, III corresponds to the teeth pairs in contact.

Simulation of the dynamic transmission error

Figures 6 and 7 show the variation of dynamic transmission errors for different values of the input torque as a function of the normalized time t / t_z, where t represents meshing time of a gear pair, and t_z is meshing time period passing the gear base pitch. In these figures, the dynamic transmission error δ_a corresponds to an input torque $T_I = 0.42$ [Nm], and δ_b is for $T_I = 0.3$ [Nm]. Evolution of the dynamic transmission error versus input torque is dependent on the number of teeth and addendum modification coefficients. An increase of the input torque leads to larger variation of the transmission error, as shown in figure 6 for gear pairs GP1 and GP2.

It should be noted that the number of teeth and addendum modification coefficients stiffness have a significant effect upon the amount and amplitude of the dynamic transmission error. The change of these geometrical parameters influence the transverse and total ratio and, therefore, the mesh stiffness characteristics are modified.

The mesh stiffness has a significant effect upon the amount and amplitude of the dynamic transmission error. For the same value of the overlap contact ratio, helical gear pairs with a smaller total contact ratio have smaller mesh stiffness and a larger variation of the dynamic transmission errors. Generally, the dynamic behaviour of helical gear pairs is improved by increasing the total contact ratio.

On the basis of these results, one can choose the best geometrical solution for minimum variation of dynamic transmission error for a given input torque, in terms of teeth number and addendum modification coefficients.

Conclusions

In this paper there are studied the dynamic characteristics of steel/plastic gear pairs with small number of pinion teeth, considered as a part of the geared servomechanism. In order to obtain gear transmissions with a high gear ratio, reduced weight and compact structures, the combination of a steel pinion with small number of teeth and a driven gear which is made from plastic has been investigated.

The specific aspects of geometrical design criteria of helical gears with small number of teeth are analyzed in order to prevent the negative phenomena of the involute tooth generation and gear engagement.

Dynamic transmission error is a result of the instantaneous contact conditions between active tooth profiles. A nonlinear helical gear model and an analytical procedure are used to calculate the dynamic characteristics of helical gears. This procedure predicts the variation of the individual dynamic factors and dynamic transmission error during the meshing cycle in relation to the geometrical and meshing characteristics of helical gear pairs.

An accurate model and the time-varying mesh stiffness as a function of the contact point along the contact line are presented in the paper. The specific geometrical design criteria for these gear pairs are taken into account in these investigations.

There were developed an interactive computational methodology and a computer program to analyze the effect of teeth number, addendum modification coefficients and input torque on dynamic transmission error of steel/plastic gears pairs with small number of teeth. The dynamic model developed in the paper takes into account the realistic mesh stiffness and should be used for more accurate design and prediction of dynamic characteristics of the steel/plastic gear pairs.

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