

Analysis of Rubber Elastic Behaviour and Its Influence on Modal Properties

GABRIEL ANGHELACHE, RALUCA MOISESCU

University Politehnica of Bucharest, 313 Splaiul Independenței, 060042, Bucharest, Romania

The paper investigates the influence of rubber elastic properties on modal behaviour of truck tyres. Hyperelastic models and numerical values of coefficients in these models used for defining rubber hyperelastic behaviour are presented. The choice of the appropriate hyperelastic model to be introduced in the tyre finite element model is motivated. The main characteristics of the finite element model of a radial truck tyre are presented. A series of modal analyses is performed, in which elastic properties of tread rubber are modified in a wide range. Finally, the influence of tread rubber elasticity characteristic values on natural frequencies and mode shapes of truck tyres is analysed.

Keywords: tyre, hyperelasticity, modal behaviour, finite element model

Rubber is one of the most important materials included in the structure of vehicle tyres. From the point of view of mechanical behaviour, elastic properties, together with hysteretic properties, definitely influence the dynamics of tyre in contact with the road.

Nowadays, a large segment of tyre research activities is developed by means of performing complex analyses on finite element models. One type of input parameters for finite element models regards the elastic properties of rubbers. In general, the strains of this type of material are large, so in many cases it is necessary to define rubber as a hyperelastic material. To define in FEA software this form of stress-strain curve, experimental results have to be obtained from tests performed on samples of each type of rubber, such an example being presented in [7]. This kind of activity is very difficult, if not impossible, for most of the researchers that do not have a direct connection with tyre manufacturing companies. An alternative source of information is the study of technical literature, but data on materials (quantitatively deficient) have to be assumed judiciously. Under these circumstances, "independent" researchers have to know and to control the magnitude of errors resulting from the analyses performed on finite element models, whereas rubber elastic properties are not known very precisely.

One of the most important analyses performed on vehicle tyres is modal analysis, from which are determined the values of resonant frequencies and mode shapes of vibration. Modal behaviour determines vibration response of tyres, particularly the "standing wave" phenomenon

occurring at high speeds of rolling and causing tyre burst [1]. Furthermore, modal behaviour contributes massively to the generation of rolling noise and influences directly the comfort of motor vehicle passengers.

In addition, modal analysis performed on finite element tyre models provides results that, compared to the experimental ones, contribute to the validation of these models from the point of view of dynamic behaviour.

This paper investigates the types of models used in finite element software to define rubber elastic behaviour. Also, the influence of rubber elastic properties on tyre modal behaviour is studied.

Analysis of Rubber Elastic Behaviour

Elastic behaviour of materials can exhibit one of the following three aspects, according to the stress-strain curve: linear elastic, nonlinear elastic or inelastic [24]. Stress-strain curves corresponding to these three types of behaviour are shown in figure 1.

Stress-strain curve shown in figure 1(a) is linear, having constant slope. This behaviour is specific to elastic materials according to Hooke's law, category including most of metals. In this case, maximum strain has small values.

A nonlinear elastic (hyperelastic) curve is shown in figure 1(b), specific to rubber-like materials. This type of material can suffer large strains (up to 500%÷1000%) and return to initial shape [14].

Loading and unloading curves are identical in case of materials whose stress-strain curves have similar shapes to those shown in figure 1(a) and in figure 1(b).

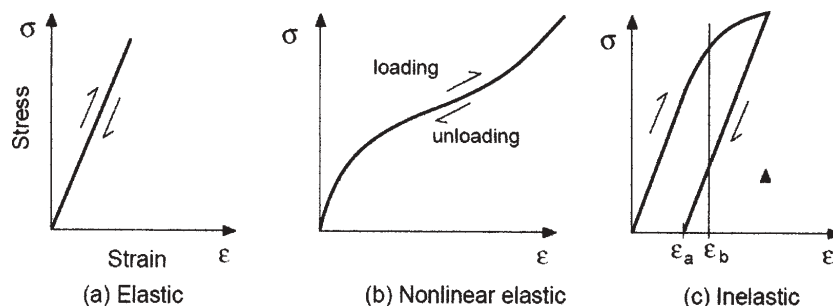


Fig. 1. Stress-strain curves representative for: a) linear elastic behaviour; b) nonlinear elastic behaviour; c) inelastic behaviour [11]

* email gab_angel@yahoo.com

In figure 1(c) the loading curve differs from the unloading curve, thus two different stress values correspond to ε_b strain. After unloading, the material maintains permanent deformation ε_a [14].

In view of defining hyperelastic properties for materials included in numerical models, finite element analysis software requires choosing the form of strain energy potential [22]. The strain energy potential defines the strain energy stored in the material per unit of reference volume (volume in the initial configuration) as a function of the strain at that point in the material [23]. The strain energy potential can be defined in various forms, such as Arruda-Boyce, Marlow, Mooney-Rivlin, neo-Hookean, Ogden, polynomial, reduced polynomial, Gent, Yeoh, Van der Waals [16]. The form is chosen taking into account the magnitude of strains in hyperelastic material, as well as the nature of stresses. For example, the neo-Hookean form is appropriate for strains up to 20÷30%, the Arruda-Boyce and Gent forms can be used for large strains (up to 300%), while the Ogden form is adequate for very large strains (up to 700%) [26].

Reference [22] expresses the strain energy potential U in the polynomial form using equations (1)÷(9):

$$U = \sum_{i+j=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (1)$$

where:

C_{ij} and D_i are temperature-dependent material parameters, \bar{I}_1 and \bar{I}_2 are the first and second deviatoric strain invariants, defined by (2)÷(5):

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \quad (2)$$

$$\bar{I}_2 = \bar{\lambda}_1^{(-2)} + \bar{\lambda}_2^{(-2)} + \bar{\lambda}_3^{(-2)} \quad (3)$$

where:

$$\bar{\lambda}_i = J^{-\frac{1}{3}} \lambda_i \quad (4)$$

$$\lambda_i = 1 + \varepsilon_i \quad (5)$$

in which ε_i are principal stretches, J is the total volume ratio, and J^{el} is the elastic volume ratio.

The elastic volume ratio J^{el} and the total volume ratio J are related by (6),

$$J^{el} = \frac{J}{J_{th}} \quad (6)$$

where J_{th} is the thermal volume strain, expressed by (7),

$$J_{th} = (1 + \varepsilon_{th})^3 \quad (7)$$

in which ε_{th} is obtained from the temperature and the isotropic thermal expansion coefficient.

Regardless of the value of N , the initial shear modulus G_0 and the bulk modulus K depend only on the polynomial coefficients of order $N=1$, through expressions (8) and (9), [22]:

$$G_0 = 2(C_{10} + C_{01}) \quad (8)$$

$$K_0 = \frac{2}{D_1} \quad (9)$$

The earliest form of the nonlinear elasticity theory, the Mooney-Rivlin form is a particular case of the polynomial form, in which $N=1$. The strain energy potential in the Mooney-Rivlin form is expressed by (10):

$$U = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + \frac{1}{D_1}(J^{el} - 1)^2 \quad (10)$$

This model provides good similarity with experimental results for rubber tension tests with stretches up to 100 %. However, the model gives less satisfactory results for compression tests and it disregards the increasing stiffness at higher strains, therefore there is no inflexion point on the stress-strain curve [25].

As stated in [22], it is justified to reduce the polynomial form, by considering that all C_{ij} with $j \neq 0$ are set to zero, which leads to the expression (11). The reduced polynomial form is obtained consequently, as a simplification of the polynomial form:

$$U = \sum_{i=1}^N C_{i0} (\bar{I}_1 - 3)^i + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (11)$$

Regardless of the value of N , the initial shear modulus G_0 depends only on the polynomial coefficient of order , through expression (12), from [22].

$$G_0 = 2 \cdot C_{10} \quad (12)$$

The Yeoh form is obtained from the reduced polynomial form for $N=3$, with the expression (13). The stress-strain curve has an inflexion point, taking into account the increasing stiffness at higher strains. This model is more versatile than the others and is adequate for cases with medium or large strains:

$$U = \sum_{i=1}^3 C_{i0} (\bar{I}_1 - 3)^i + \sum_{i=1}^3 \frac{1}{D_i} (J^{el} - 1)^{2i} \quad (13)$$

The neo-Hookean form is the particular case of reduced polynomial form with $N=1$. The strain energy potential in the neo-Hookean form is expressed by (14):

$$U = C_{10}(\bar{I}_1 - 3) + \frac{1}{D_1}(J^{el} - 1)^2 \quad (14)$$

According to [3], rubber can be considered an incompressible material. For such materials, the last term in the strain energy potential expression is equal to zero. Therefore, the simplest form of the neo-Hookean model is (15).

$$U = C_{10}(\bar{I}_1 - 3) \quad (15)$$

The stretches of rubber constituents in motor vehicle tyres are small [3]. Therefore, the neo-Hookean form is considered for the finite element model of truck tyre. An identical approach is found in finite element modelling of a radial passenger car tyre, illustrated in [21].

Properties of Rubber Used in Tyre Modelling

Finite element analysis software, such as [20] and [26], provides two approaches for defining hyperelastic properties of materials included in numerical models:

- input of tabular experimental results;
- input of values for coefficients of the chosen form of strain energy potential.

One of the objectives of this paper is to review numerical values mentioned in literature for the parameters defining elastic properties of rubbers constituents in finite element models of motor vehicle tyres. In all cases, the rubber was considered incompressible, thus the last term (D) in the strain energy potential expression is equal to zero. These values are presented in table 1, in which the calculated values are mentioned in brackets.

Table 1
NUMERICAL VALUES FOR ELASTIC PROPERTIES OF RUBBER USED IN TYRE MODELLING

Reference	Modulus		Model	$G_0 = 2 C_{10}$ (reduced polynomial model)		$G_0 = 2 (C_{10} + C_{01})$ (polynomial model)	
	E [MPa]	G_0 [MPa]		C_{10} [MPa]	C_{20} [MPa] C_{30} [MPa] C_{40} [MPa]	C_{10} [MPa]	C_{01} [MPa]
Abaqus [22]		(2)	Neo-Hookean	1			
Olatunbosun, Bolarinwa [4], [16]		(2)	Neo-Hookean	1			
Wang, Lu [18]		(0,7949)	Neo-Hookean	0,3974			
Wang, Lu [18]		(0,8942)	Mooney-Rivlin	(0,4471)		0,2856	0,1615
Lin, Hwang [13]		(0,0942)	Mooney-Rivlin	(0,0471)		0,1189	-0,0718
Yanjin, et al. [17]		(1,1094)	Mooney-Rivlin	(0,5547)		0,5510	0,0037
Altidis, Adams [2]		(0,756÷1,84)	Mooney-Rivlin	(0,378÷0,92)		0,302÷0,736	0,076÷0,184
Mars [15]		(1,984)	Reduced polynomial	0,992	0,041 0 -1,8·10 ⁻⁴		
Castellucci, et al. [5]		(2,652)	Yeoh	1,326	-0,326 0,132		
Hofstetter, et al. [10]		(0,7588)	Yeoh	0,3794	0,0232 -0,0003		
Kim, Noor [12]		1,041	-				
Ciullo, Hewitt [6]	14,9 15,4 15,6	(4,97) (5,13) (5,2)	-	(2,483) (2,567) (2,6)			
Beatty [3]	2,9÷4,1 (static - tread band rubber)	1,0÷1,4 (static - tread band rubber)		(0,5÷0,7)			

Observations

In table 1 are included numerical values of rubber elastic properties from two sources: hyperelastic models and initial elasticity.

All hyperelastic models in the cited references have reduced polynomial form (with $N=1$ or $N=3$, but also one with $N=4$) or polynomial form (with $N=1$). Models with $N=1$ are predominant (neo-Hookean and Mooney-Rivlin).

For all hyperelastic models, the values of coefficient C_{10} range from about 0.38 MPa to 1.33 MPa.

The values stated in paper [13] were not taken into account, as they differ by an order of magnitude from all the others.

Information regarding initial elastic behaviour indicates higher values of initial shear modulus G_0 and Young's modulus E compared to the values calculated from the information in hyperelastic models.

Information on hyperelastic model type and numerical values of rubber elastic properties are used for developing the finite element model of truck tyre.

Finite Element Model of Truck Tyre

The finite element model of a radial truck tyre is developed in view of theoretical investigations on modal behaviour. The model is created in Abaqus FEA software.

The structure of tyre finite element model is nonlinear and anisotropic related approaches being mentioned in [9, 11]. The structure and properties of different components of real tyre are accurately reproduced. Mesh definition of tyre section model takes into account the layout of different types of rubber and cord plies with various orientations.

Rubber of treadband, undertread area, sidewalls and bead filler are defined as incompressible hyperelastic materials. According to previous chapters, for the finite element model of truck tyre the neo-Hookean form is considered. For polyamide and steel cord plies, the materials are defined as linear elastic isotropic.

The 2D model of tyre section includes two types of elements: solid (for rubber constituents) and surface elements (for cord plies and bead wire) [19]. All surface elements are embedded in solid elements and have rebar layers, for which material, section area, distance between wires and angle are defined, according to properties of cord in truck tyre design. A related approach is presented in [8].

Radial truck tyre structure is symmetrical, except for a steel cord layer in the breaker. Therefore, it is compulsory to model an entire tyre section, which requires additional computational resources.

Tread design of real tyre has quasi-repetitive triangular

shape in circumferential direction. Modelling the real profile of treadband generates a significant increase in the number of finite elements, which may prevent the analysis from being performed, because of limited computational resources. The current model takes into consideration circumferential grooves, but with a simplified (rectilinear) shape.

The 3D tyre model, obtained from the 2D model by rotation, is shown in figure 2. Mesh density is uniform along the tyre circumference. Also, the circumferential mesh is relatively refined, in view of increasing the precision of modal analysis. Tyre support by the rim is modelled by constraining the nodes in the bead area that come into contact with the rim.

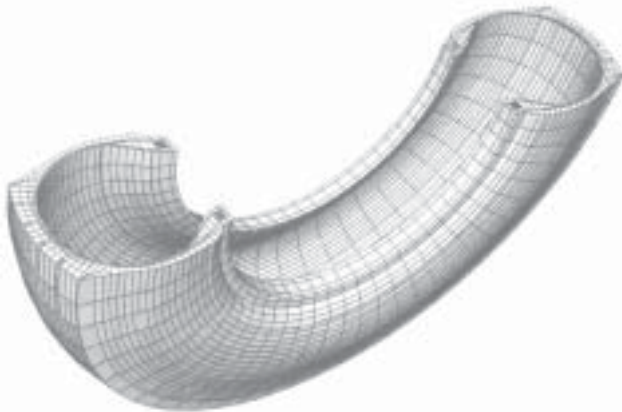


Fig. 2. Sectioned model of truck tyre

The following analyses are performed on the finite element model of truck tyre, without road contact: inflation and modal analysis. The shape of tyre model obtained from inflation analysis agrees to the one obtained experimentally.

Modal analysis provides results regarding tyre natural frequencies, as well as corresponding mode shapes of vibration. Following the analysis of results obtained, it is observed that in the range of small frequencies, between 0 Hz and 240 Hz, the tyre has dozens of resonances in radial and lateral directions, but also combinations of these

resonances. In figures 3 and 4 are shown truck tyre mode shapes of order 1, 2, 3 and 4, radial and lateral.

Influence of Elastic Properties of Tread Rubber on Tyre Modal Behaviour

One of the difficulties that researchers are facing in finite element tyre modelling is determining the hyperelastic properties of rubbers used in tyre design. The best way of determining these properties is to perform experimental research on samples from each type of rubber. Unfortunately, this method requires specialized research equipment and high costs. Researchers from tyre manufacturing companies have the necessary experimental data, but these are not available outside the companies. Another method consists of consulting the existing literature, in which insufficient information is available, mostly regarding the shear modulus and Young's modulus, but also some values of C_{10} used within neo-Hookean models. With this information it is not possible to describe hyperelastic behaviour of rubber for large strains (> 300%). However, for tyre models, whose rubbers suffer small strains, using such data provides adequate accuracy.

From the analysis of shear modulus values for different types of rubber, mentioned in literature and gathered in table 1, it results that, excepting for [3], the values of shear modulus range from 0.75 MPa to 5.2 MPa, while the values of C_{10} coefficient range from 0.38 MPa to 1.33 MPa.

In order to highlight the influence of the values of C_{10} coefficient on natural frequencies of the modelled tyre, a series of modal analyses was performed on the tyre model, in which the values of coefficient for tread rubber were modified sequentially. The range of values for this coefficient is from 0.3 MPa to 1.9 MPa. The results obtained are shown in figure 5.

Figure 5 shows that increasing the C_{10} coefficient leads to a small increase in natural frequencies, both in radial and lateral direction. The increase of natural frequencies is not linear in respect to C_{10} , as the rate of increase is higher at smaller values of C_{10} , between 0.3 MPa and 1.1 MPa. The first three resonance frequencies in radial direction are modified by less than 1 Hz for the entire range

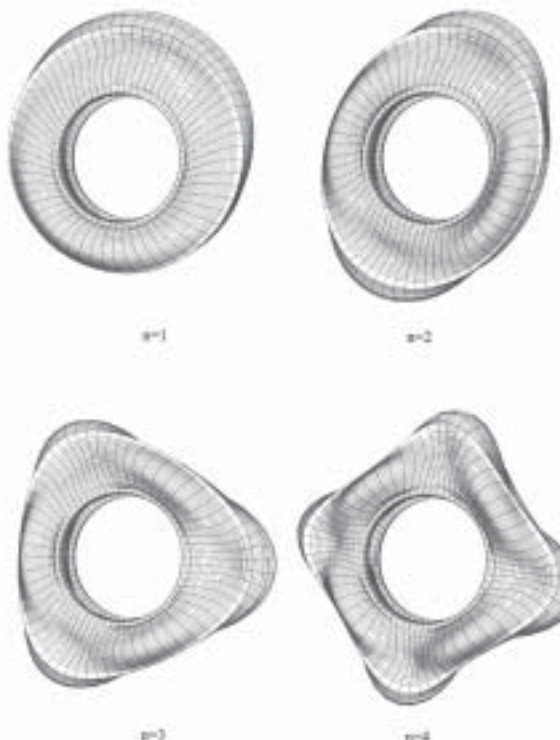


Fig. 3. Radial mode shapes of order 1,2,3 and 4

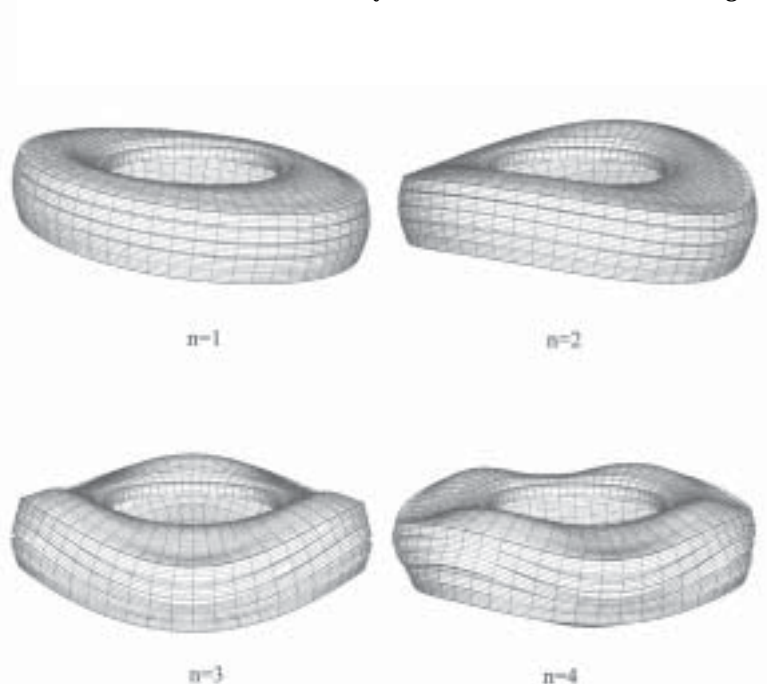


Fig. 4. Lateral mode shapes of order 1,2,3 and 4

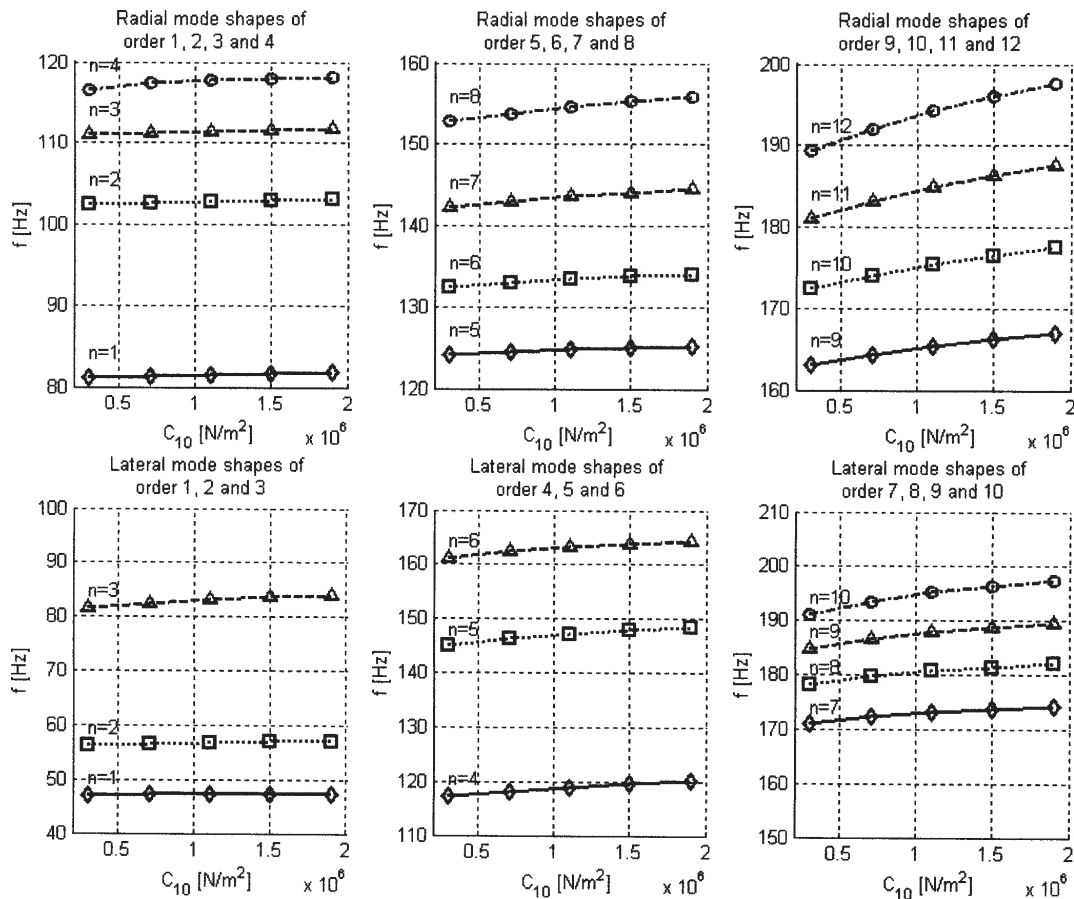


Fig. 5. Influence of C_{10} coefficient on radial and lateral natural frequencies of modelled tyre

of variation of C_{10} . The next three resonance frequencies in radial direction are modified by less than 2 Hz. The resonance frequencies in lateral direction are modified by less than 1 Hz only for orders 1 and 2. For orders starting at 3, it results that resonance frequencies in lateral direction suffer higher variations than the radial ones of the same order.

As the order of resonance frequencies (both radial and lateral) is higher, the increase in their values is more pronounced as the C_{10} coefficient raises. As an example, for order 10, the increase of resonance frequency is 5.13 Hz in radial direction and 6.21 Hz in lateral direction. Expressed in values relative to the corresponding frequencies for medium rubber stiffness, for the same order 10, the increase of frequency is 2.92% in radial direction and 3.18% in lateral direction.

Conclusions

The accuracy of analyses performed using FEA software depends, among other things, on elastic properties of rubbers in tyre model design. Rubber elastic behaviour is nonlinear (hyperelastic). There are several types of models that characterize hyperelastic behaviour. Within this paper were presented the most important forms of strain energy potential. The model of hyperelastic behaviour can be chosen according to experimental data. If such information is not available, the choice is made by taking into account the magnitude of stretches suffered by the rubber. For tyre models, rubber stretches are small, therefore a simple model, such as neo-Hookean, can be chosen. It was noticed that in literature there is a large variation of values of C_{10} coefficient mentioned for tyre rubbers.

To emphasize the influence of these values on natural frequencies of 3D truck tyre model, a series of modal analyses was performed, in which the values of C_{10}

coefficient were modified successively for tread rubber. It was observed that when the C_{10} coefficient increases, there is a small increase in the values of resonant frequencies. The higher is the order of resonant frequencies, the more pronounced is the increase in their values, as the C_{10} coefficient raises.

Increasing the stiffness of tread rubber by 533% leads to an increase smaller than 3.2% of natural frequencies values up to order 10, both in radial and lateral direction.

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