

The Heat Transport in Nanofluids: a Theoretical Approach Through the Fractal Theories

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A mathematical model of the heat transport in nanofluids using the fractal theories (the scale relativity theory in the topological dimension $DT = 2$) is established. Through a scale covariance form of the Newton's equation, a Navier-Stokes type equation with an imaginary viscosity coefficient and particularly a Schrödinger's type equation are obtained. Some applications of the model are given like the heat transport through an effective thermal conductivity, by Brownian motion or by liquid layering at liquid-nanoparticle interface. It results that the heat transport in nanofluids is performed through an unique mechanism, the above standard sequences mentioned being imposed by the interaction scales. Moreover, the quantum thermal conductance of electrons in an one-dimensional wire is obtained.

Keywords: nanofluid, heat transfer, fractal, thermal conductance

Nanofluid is a new kind of heat transfer medium containing nanoparticles which are uniformly and stable distributed in a base fluid. Experiments on nanofluids have demonstrated that the thermal conductivity increases with grain size decreasing [1-4]. Keblinski *et al* [4] have examined four possible mechanisms for the anomalous enhancement observed in nanofluids: Brownian motion of the nanoparticles [5-7], molecular-level layering of the liquid at the liquid-nanoparticles interface [8], the effects of nanoparticle clustering [9], and ballistic phonons transport [4].

Recently, the increasing of the heat transfer in nanofluids was related to the fractal effects [10, 11]. Moreover, Wang *et al.* [12] reported that the modified fractal model agreed well with the experimental data obtained for the SiO₂/ethanol nanofluid. In such conjecture, the fractal theories (particularly the scale relativity theory (SRT) [13, 14]) is a new approach to understand quantum mechanics, and furthermore physical domains involving scale laws, such as the nanosystems [15, 16]. It is based on a generalization of Einstein's principle of relativity to scale transformations. Namely, one redefines space-time resolutions as characterizing the state of scale of reference systems, in the same way as velocity characterizes their state of motion. Then it requires that the laws of physics apply whatever the state of the reference system, of motion (principle of motion-relativity) and of scale (principle of SRT). The principle of SRT is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolution. For example, considering that the motion of micro-particles take place on continuous but non-differentiable curves, *i.e.* on fractals [13, 14], it was demonstrated that, in the topological dimension [17] $D_T=2$, the geodesics of the fractal space-time are given by a Schrödinger's type equation.

In the present paper, using the SRT, we propose a new mechanism capable to explain the experimentally observed enhanced thermal conductivity of nanofluids. In such conjecture, the heat transport through an effective thermal conductivity, by Brownian motion or by liquid

layering at liquid-nanoparticle interface is performed through an unique mechanism imposed by the interaction scales.

Theoretical part

Mathematical model

A non-differentiable continuum is necessarily fractal and the trajectories in such a space (or space-time) own (at least) the following three properties:

- the test particle can follow an infinity of potential trajectories: this leads to the use of a fluid-like description;
- the geometry of each trajectory is fractal (of dimension 2 – for other details on the fractal dimension see [17]). Each elementary displacement is then described in terms of the sum, $dX = dx + d\zeta$, of a mean classical displacement $dx = vdt$ and of a fractal fluctuation $d\zeta$, whose behavior satisfies the principle of SRT (in its simplest Galilean version). Hence $\langle d\zeta \rangle = 0$ and $\langle d\zeta^2 \rangle = Ddt$ where D defines the fractal/non-fractal transition, *i.e.* the transition from the explicit scale dependence to scale independence and c is the light speed in vacuum. The existence of this fluctuation implies introducing new third order terms in the differential equation of motion;

- time reversibility is broken at the infinitesimal level: this can be described in terms of a two-valuedness of the velocity vector for which we use a complex representation, $V = (v_+ + v_-)/2 - i(v_+ - v_-)/2$. We denoted v_+ the "forward" speed and v_- the "backward" speed.

These three effects can be combined to construct a complex time-derivative operator.

$$\frac{\delta}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla - iD\Delta \quad (1)$$

Now, the first Newton's principle in its covariant form $\delta V / dt = 0$, becomes.

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + V \cdot \nabla V - iD\Delta V = 0 \quad (2)$$

i.e. a Navier-Stokes type equation in a fractal space-time. This means that, both for the differential scale and the

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fractal one, the complex acceleration field, $\delta V / dt$, depends on the local time dependence, δV , on the non-linearity (convective) term, $V \cdot \nabla V$, and on the dissipative one, ΔV . Moreover, the behaviour of a “non-differentiable fluid” is viscoelastic or hysteretic type. Such a result is in agreement with the opinion given in [15, 16, 18]: the non-differentiable fluid can be described by Kelvin-Voight or Maxwell rheological model with the aid of a complex quantities e.g. the complex speed field, the complex acceleration field and with the complex structure coefficients e.g. the imaginary viscosity coefficient.

We note that the using of a complex quantities or a complex structure coefficients are compatible with the fractal structure of space-time [13, 14].

From (2) and by the operational relation $V \cdot \nabla V = \nabla(V^2/2) - V \times (\nabla \times V)$ we obtain the equation:

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - V \times (\nabla \times V) - iD\Delta V = 0 \quad (3)$$

If the motions of the “non-differentiable fluid” are irrotational, i.e. $\Omega = \nabla \times V = 0$ we can choose V of the form:

$$V = \nabla \phi \quad (4)$$

with ϕ a complex speed potential. Then, equation (3) becomes:

$$\frac{\delta V}{dt} = \frac{\partial V}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - iD\Delta V = 0 \quad (5)$$

and more, by substituting (4) in (5), we have

$$\nabla \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 - iD\Delta \phi \right) = 0 \quad (6)$$

This yield:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 - iD\Delta \phi = F(t) \quad (7)$$

with $F(t)$ a function of time only. We realize that (5) have been reduced to a single scalar relation (7), i.e. a Bernoulli-type equation.

Let us choose ϕ of the form

$$\phi = -2iD \ln \psi \quad (8)$$

Then, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ e.g. $F(t) \equiv 0$, the (7) becomes a Schrödinger type equation:

$$D^2 \Delta \psi + iD \partial_t \psi = 0 \quad (9)$$

Now, the relation

$$\frac{\partial \phi}{\partial \ln \psi} = -2iD \quad (10)$$

is fixing the interaction scale and the heat transport type respectively.

In the following, some applications of the model are given.

Results and discussion

Heat transport through an effective thermal conductivity

For $D = -i\chi$ and $\psi = \ln(T/T_0)$ the (9) is reduced to a diffusion equation in terms of the normalized temperature field T/T_0 , i.e.

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T \quad (11)$$

χ is the effective thermometric conductivity defined as

$$\chi = k / \rho c_p \quad (12)$$

with k the effective thermal conductivity, ρ the “density” of the composite, and c_p the “specific heat” of the composite. For details on the effective thermal conductivity, density of the composite etc., see the paper [19].

Particularly, in the Hamilton and Crosser (HC) model [20], the expression for the effective conductivity of matrix (in our case the liquid) that contains a dispersion of particles has the form

$$k_{HC} = k_f \left[\frac{k_p + (n-1)k_f - (n-1)V_p(k_f - k_p)}{k_p + (n-1)k_f + V_p(k_f - k_p)} \right] \quad (13)$$

In this relation k_{HC} is the predicted thermal conductivity of the composition system, k_f is the conductivity of the liquid, k_p is the conductivity of the solid particles, V_p is the particle volume fraction, and n is the empirical shape factor (e.g. for spheres $n=3$). Some consequences result:

- the particle size does not play a role and only the particle shape and volume fraction affect thermal conductivity (13). This is inconsistent with the above mentioned experimental results [1-3], which demonstrate that, for example, at the same volume fraction, k_{eff} is greater for 15 nm particles than for 40 nm particles;

- for a given volume fraction (assuming spherical particles for simplicity), the maximum conductivity is achieved when $k_p \gg k_f$ and is given by

$$k_{HC} = k_f \left(\frac{1 + 2V_p}{1 - V_p} \right) \quad (14)$$

which is the upper limit of the thermal transport enhancement within the macroscopic theory. Also, according to [4] “a closer examination of (13) shows that particles of $k_p = 10k_f$ or greater lead to an increase of k_{HC} close to the limiting value given by (14); surprisingly, any further increase of k_p has little effect on the thermal conductivity of the composite. Physically, this behavior arises because with significantly (10 times or more) higher particle thermal conductivity, the temperature across the particle becomes essentially constant, thereby providing the same boundary condition for the heat-flow equation in the surrounding liquid. Because the thermal conductivity of nearly all crystalline solids are at least ten times those of liquids, the above analysis would suggest that the thermal conductivity of nanofluids should be independent of particle composition; experimentally, this is clearly not the case. Moreover, the HC limit can be greatly exceeded, as exemplified by measurements of thermal conductivity in Cu nanofluids, in which $\kappa > 10$.”

For an extended analysis, it is useful to introduce the excess-thermal-conductivity-enhancement coefficient κ , defined as

$$\kappa = \frac{k_{eff} - k_f}{k_{HC} - k_f} \quad (15)$$

In the above definition, κ is simply the ratio of measured thermal conductivity increase divided by the increase predicted by the HC model. Consequently, $\kappa=1$ indicates agreement with the macroscopic theory, and $\kappa > 1$ measure the magnitude of thermal-conductivity enhancement.

Other models with effective thermal conductivity are given in [21-23].

Heat transport through the Brownian motion

For $D = -i\mathcal{D}_0$ and $\psi = \ln(\rho/\rho_0)$ the (9) is reduced to a diffusion equation in terms of the normalized density field $\rho > \rho_0$, i.e.

$$\frac{\partial \rho}{\partial t} = \mathcal{D}_0 \nabla^2 \rho \quad (16)$$

In such conjecture the theory of the random movement, i.e. the Brownian motion, is assimilated with a diffusion process [24- 26].

Brownian motion is characterized by the particle diffusion constant \mathcal{D}_0 , given by the Stokes-Einstein formula

$$\mathcal{D}_0 = \frac{k_B T}{3\pi\eta d} \quad (17)$$

where k_B is the Boltzmann constant, η is the fluid viscosity and d is particle diameter. We note that the relation (17) is directly obtained in the random movement analysis.

With (17), one can estimate the effect of Brownian motion on the thermal conductivity by comparing the time scale of nanoparticle motion with that of heat diffusion in the liquid. Equivalently we can compare the time required for a particle to move by the distance equal to its size τ_D , given by [4]

$$\tau_D = \frac{d^2}{6\mathcal{D}_0} = \frac{3\pi\eta d^3}{6k_B T} \quad (18)$$

with time required for heat to move in liquid by the same distance τ_H ;

$$\tau_H = \frac{d^2}{6\chi} = \frac{d^2 c_p \rho}{6k_f} \quad (19)$$

For water at room temperature [4], ($\eta = 0.01$ g/cm . s, $k_f = 0.4$ W / m . K, $\rho = 1$ g / cm³, $c_p = 4.2$ J / g) and with $d = 10$ nm, (18) and (19) give $\tau_D \approx 2 \cdot 10^{-7}$ s and $\tau_H \approx 4 \cdot 10^{-10}$ s, respectively. The ratio of τ_D / τ_H is ≈ 500 and decreases to ≈ 25 when the particle size is equal to the atomic size (≈ 0.5 , demonstrating that the thermal diffusion is much faster than Brownian diffusion, even within the limits of extremely small particles.

The above comparison demonstrates that the movement of nanoparticles due to Brownian motion is too slow to transport significant amounts of heat through a nanofluid, a conclusion supported by the results of molecular level simulations [24, 25].

Heat transport through liquid layering at liquid-nanoparticle interface

Let us consider the interaction between two fractal structures, e.g. the fluid and the nano-particle [10, 11], and the corresponding interface. According with the SRT [13, 14], the interface dynamics at the mesoscopic scale [15, 16], $h = 2mD$, is described by the coupled equations set

$$\begin{aligned} i\hbar\partial_t \Psi_f &= H_f \Psi_f + H_{f-np} \Psi_{np} \\ i\hbar\partial_t \Psi_{np} &= H_{np} \Psi_{np} + H_{f-np} \Psi_f \end{aligned} \quad (20a,b)$$

with Ψ_{np} , Ψ_{np} the wave functions, H_f , H_{np} the Hamiltonians on either side of the interface, $H_{f, np} = \text{const.}$ the Hamiltonian of the interface, \hbar the reduced Planck constant and the indices (f, np) refers to the fluid and nanoparticle, respectively. The Hamiltonians H_f correspond to the free energy of the fluid, H_{np} to the the free energy of the nanoparticle and $H_{f, np}$ to the free energy of the interface [11, 13, 14].

Expliciting the wave functions by the following relations:

$$\Psi_f = \sqrt{\rho_f} e^{i\theta_f}, \quad \Psi_{np} = \sqrt{\rho_{np}} e^{i\theta_{np}} \quad (21a,b)$$

and separating in (20a, b) the real parts from the imaginary ones, we obtain:

$$\begin{aligned} \partial_t \rho_f &= -\partial_t \rho_{np} = \frac{H_{f-np}}{\hbar} \sqrt{\rho_f \rho_{np}} \sin(\theta_{np} - \theta_f) \\ \partial_t \theta_f &= -\frac{H_f}{2\hbar} - \frac{H_{f-np}}{2\hbar} \sqrt{\frac{\rho_{np}}{\rho_f}} \cos(\theta_{np} - \theta_f) \\ \partial_t \theta_{np} &= -\frac{H_{np}}{2\hbar} - \frac{H_{f-np}}{2\hbar} \sqrt{\frac{\rho_f}{\rho_{np}}} \cos(\theta_{np} - \theta_f) \end{aligned} \quad (22a-c)$$

where $\sqrt{\rho_f}$, $\sqrt{\rho_{np}}$ are the amplitudes and θ_{np} , θ_{np} the phases. From here, with [4, 8]

$$\begin{aligned} \rho_n &= \rho_{np} = \rho, \quad H_f = k_B T_f, \quad H_{np} = \\ &= k_B T_{np}, \quad \Delta T = T_f - T_{np}, \quad \Delta\theta = \theta_{np} - \theta_f \end{aligned} \quad (23a-e)$$

it results the heat flux:

$$J_Q = \varepsilon(\partial_t \rho_f - \partial_t \rho_{np}) = J_{Q,M} \sin \Delta\theta \quad (24)$$

of amplitude $J_{Q,M}$

$$J_{Q,M} = \frac{2\varepsilon H_{f-np}}{\hbar} \rho \quad (25)$$

and phase difference θ

$$\theta = \theta_0 + \frac{k_B}{\hbar} \int \Delta T dt, \quad \theta_0 = \text{const.} \quad (26a,b)$$

with T_f , T_{np} the absolute temperatures, ΔT temperature difference on the interface, ε the elementary amount of energy transferred through the interface [9] and k_B the Boltzman's constant.

For $\Delta T = 0$, relation (26a, b) reproduces a d.c. Josephson effect of thermal type, while for $\Delta T \neq 0$ an a.c. Josephson effect of thermal type, i.e. oscillations of the heat flux with the pulsation,

$$\omega = \frac{k_B \Delta T}{\hbar} \quad (27)$$

In this last case, let us consider the dependency $\Delta T = T(t)$ in the form

$$\Delta T = T_0 + \bar{T}_0 \cos(\Omega t + \varphi_0), \quad \varphi_0 = \text{const.} \quad (28a,b)$$

We notice that any time-dependent signal, e.g. (28a,b) admits locally a Fourier discrete decomposition [27]. This means that the previous results are of maximum generality.

Substituting the relation (28a,b) into (26a,b) and integrating it, we obtain the time dependence of the phase difference:

$$\Delta\theta = \theta_0 + \frac{k_B T_0}{\hbar} t + \frac{k_B \bar{T}_0}{\hbar \Omega} \sin(\Omega t + \varphi_0), \quad \theta_0 = \text{const.} \quad (29a,b)$$

With relation (29a,b), the expression (24) of the heat flux becomes

$$\begin{aligned} J_Q(t) &= J_{Q,M} \sin \left[\theta_0 + \frac{k_B T_0}{\hbar} t + \frac{k_B \bar{T}_0}{\hbar \Omega} \sin(\Omega t + \varphi_0) \right] = \\ &= J_{Q,M} \sum_{n=-\infty}^{+\infty} (-1)^n J_n \left(\frac{k_B \bar{T}_0}{\hbar \Omega} \right) \sin \left[\left(\frac{k_B T_0}{\hbar} - n\Omega \right) t + \phi_0 \right] \end{aligned} \quad (30)$$

where J_n is the n-order Bessel function and ϕ_0 a constant of integration.

When the pulsation $\Omega_n = k_B T_0 / \hbar$ satisfies the relation

$\Omega = n\Omega = n=1,2,3,\dots$, the time-average of $J_Q \approx \langle J_Q(t) \rangle$ differs from zero, *i.e.* there is a continuous component of the heat flux of the form:

$$J_{Q,C} = (-1)^n J_{Q,M} J_n \left(\frac{k_B \bar{T}_0}{\hbar \Omega} \right) \sin \phi_0 \quad (31)$$

From relation (31) peaks of the continuous heat flux result for

$$T_n = nT_c, \quad T_n = T_0, \quad T_c = \frac{\hbar \Omega}{k_B}, \quad n=1,2,3,\dots \quad (32a-d)$$

and consequently a negative differential thermal conductance ($dJ_{Q,C}/dT_p < 0$). Moreover, from equation (31) the heat flux of the pick n can vary continuously in the range $[-J_{Q,M} J_n(k_B \bar{T}_0 / \hbar \Omega), +J_{Q,M} J_n(k_B \bar{T}_0 / \hbar \Omega)]$ at constant temperature T_n , and the phase varies in the range $[-\pi/2, +\pi/2]$. This means that in the interface, the heat can be generated or absorbed. When the interface generates the heat, the increase of the thermal conductivity of the nanofluid can be explained.

In the general cases [4], the heat transfer in nanofluids is non-linear, while, in the linear approximation *i.e.* $\sin \Delta\theta \approx \Delta\theta = k_B \tau \Delta T / \hbar$, the standard form of heat transfer results,

$$J_Q \approx G \Delta T, \quad G = J_{Q,M} \frac{k_B \Delta \tau}{\hbar} \quad (33a,b)$$

with G the thermal conductance and $\Delta \tau$ a characteristic time [1-4].

Moreover, by an extension of the model to heat transport in an one-dimensional wire, the relation (33b) with the restrictions [15, 16, 28, 29] $\varepsilon = k_B T / 3, 8H_{fnp} \rho \tau / \hbar = 1$ implies the quantum thermal conductance of electrons [30], $G_e = \pi^2 k_B T / 3\hbar$.

Conclusions

A mathematical model of the heat transport in nanofluids using the scale relativity theory in the topological dimension $D_T = 2$ is established. By means of a scale covariance form of the Newton's equation, a generalized Navier-Stokes type equation with an imaginary viscosity coefficient is obtained. Then, the macroscopic behaviour of the nanofluids is viscoelastic as or hysteretic type.

A Schrödinger type equation is obtained as an irrotational movement of the nanoparticles.

Some applications of the model are given as the heat transport through an effective thermal conductivity, by Brownian motion or by liquid layering at liquid-nanoparticle interface. The heat transport in nanofluids is performed through an unique mechanism, the above standard sequences mentioned being imposed by the interaction scales.

By an extension of the model to heat transport in an one-dimensional wire, the quantum thermal conductance of electrons is given.

Acknowledgement: We wish to thank to Professor S.U.S. Choi for helpful discussion on the manuscript of this paper.

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Manuscript received: 11.12.2007