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Study of the Vibrations of Some Composite Bars with Polypropylene Honeycomb Core and Carbon Fiber and Fiberglass Fabric Faces

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Abstract*. The paper generalizes the Timoshenko model for thick bars, using a new model that is applied to the vibration study of multilayer composite bars. In the proposed mathematical model, three coefficients are introduced that take into account the non-uniformities of the tangential and normal stresses in the bar section. The vibrations of some composite bars with a polypropylene honeycomb core with a thickness of 10 mm, 15 mm and 20 mm are experimentally studied, on the faces of which one or two layers of carbon fiber, respectively glass fiber was poured. For each analysed bar, the stiffness and the equivalent modulus of elasticity are determined and the variation of the damping coefficient according to the length of the bar is studied.*

Keywords*: polypropylene honeycomb, composite bars, damping factor, vibration frequency*

1. Introduction

A special place in the study of the dynamics of composite materials was occupied by sandwich bars consisting of several layers, with constant thickness, superimposed. Most studies refer to sandwich bars made of three layers, the middle layer of a viscoelastic behaviour, the lower and upper layers having superior elastic and strength properties.

Most of the studies were based on the following assumptions about the behaviour of sandwich laminates:

- there is a continuity of displacements and stresses on the separating surfaces between the layers;
- the forces of transversal inertia predominate, neglecting the longitudinal inertia and the rotational inertia of the bar sections;
- there are no deformations on the thickness of the bar, so the transverse deformations are the same on the whole section of the bar;
- the core has an elastic or viscoelastic behaviour taking over the shear stresses (tangential);
- the core is not subjected to normal stresses;
- the outer layers have an elastic behaviour, being required for pure bending.

Higher order deformation theories take into account the deformation of the cross sections and satisfy the conditions for the cancellation of the tangential stresses on the outer surfaces, without a shear correction factor. In the literature, various higher-order shear theories that meet the above conditions are proposed by several researchers. These theories differ, in particular, in the functions that appreciate the variation of tangential stresses on the bar section. The most well-known higher order deformation theories are based on the following types of functions:

- parabolic, symbolized PSDBT (parabolic shear deformation beam theory), introduced by [1];
- trigonometric, symbolized TSDBT (trigonometric shear deformation beam theory), introduced by [2];
- hyperbolic, symbolized HSDBT (hyperbolic shear deformation beam theory), introduced by [3];

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- exponential, symbolized ESDBT (exponential shear deformation beam theory), introduced by [4].

A new higher-order distortion theory, symbolized by ASDBT, was proposed more recently by [5], and a comparison of the results of these theories was made by [6] and later by [7]. Although in the literature there are numerous studies on isotropic laminated composite beams [8-18], not all aspects of mechanical behaviour are elucidated. A special case is hosted by functional graded materials (FGM-as Functionally graded material) are special composites whose mechanical properties vary on their thickness.

A general method for calculating the elastic properties of these materials is presented in [19]. A study of these materials, based on the Euler-Bernoulli theory, is presented in [20], in which it is assumed that Young's modulus varies exponentially in thickness. In [21] the free vibrations of a simply supported FGM bar are studied, also using the Euler-Bernoulli theory, but considering tangential shear stresses which in section have a parabolic and exponential variation, respectively. In [22] the fundamental frequencies of FGM bars with different boundary conditions are analysed using the classical theories of first-order shear (FSDT), respectively of higher order (HSDT). The results are subsequently generalized by [23] and for non-classical boundary conditions, examining the influence of the material parameters and the arrangement of the layers on the slenderness coefficients of the bars.

Recent applications have shown that honeycomb-reinforced fiberboard panels can be used for new construction or for refurbishing existing structures. In [24] the structural vibrations of honeycomb sandwich with sinusoidal core geometry are studied. A higher order model for vibration studies has been developed, formulated with energy methods. A special problem concerns the vibration damping of multilayer composite materials. Results in this regard are obtained in [25-27].

In [28] the free vibrations of curved sandwich bars with flexible core under different temperature conditions were studied. The core and outer faces were considered to be made of materials with temperature-dependent mechanical properties. There is a vibration that the free frequency of the bars decreases if the temperature rises.

This paper studies the vibrational behavior of sandwich type composites with carbon fiber fabric / fiberglass fabric faces. Carbon fiber composites offer unique properties, including high stiffness and low weight, near zero thermal expansion and radiolucency. Carbon fiber composites do not block X-rays, allow a short scan time with precise results, and prevent excessive exposure to X-rays. Instead, glass can be used to protect against radiation exposure. In addition to its adjustable mechanical, chemical and optical properties, glass can substantially absorb gamma radiation, X-rays and neutrons.

2. Materials and methods

2.1. Theoretical considerations

It is considered a wide sandwich bar b , with geometric and mass symmetry with respect to the middle layer (Figure 1).

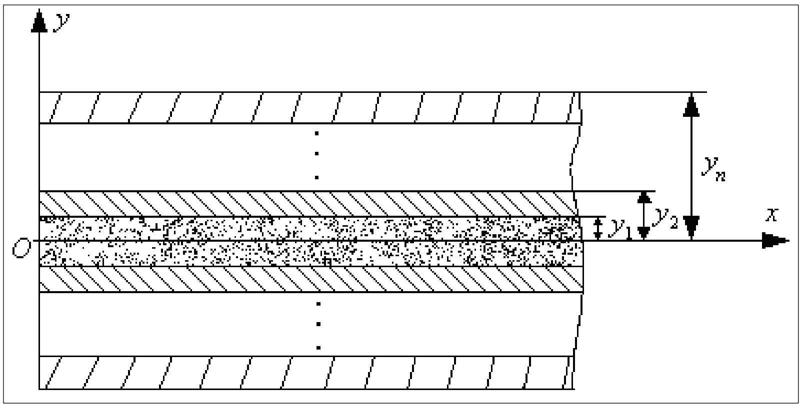

Figure 1. Sandwich bar with geometric and massive symmetry to the middle layer.

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It is noted with y_k, $k=\overline{1,n-1}$, the abscissa of the separation surface between the layers k and $k+1$ and with y_n the abscissa of the outer surface of the layer n. Due to the symmetry, it is highlighted deformations of the base for $y \geq 0$. It is considered that the displacements along the axes Ox and Oy, for k the layer, are of the form:

$$\begin{aligned} u_x^{(k)}(x,y,t) &= \left(y \quad y^3\right) \cdot \mathrm{B}_k \cdot \alpha(x,t) - y \cdot \frac{\partial w(x,t)}{\partial x}, \\ u_y^{(k)}(x,y,t) &= w(x,t),\; k=\overline{1,n}, \end{aligned} \tag{1}$$

with:

$$\mathrm{B}_n = \begin{pmatrix} 1 \\ -\dfrac{1}{3y_n^2} \end{pmatrix}, \tag{2}$$

but:

$$\mathrm{B}_k = \mathrm{A}_{k,k+1} \cdot \mathrm{B}_{k+1},\; k=\overline{1,n}, \tag{3}$$

where $\mathrm{A}_{k,k+1}$ this is the compatibility matrix between the stresses, respectively the k layer deformations and the stresses, respectively the $k+1$ layer deformations. These consist of:

$$\mathrm{A}_{k,k+1} = \begin{pmatrix} \dfrac{3G_k - G_{k+1}}{2G_k} & \dfrac{3G_k - 3G_{k+1}}{2G_k} \cdot y_k^2 \\ \dfrac{G_{k+1} - G_k}{2G_k y_k^2} & \dfrac{3G_{k+1} - G_k}{2G_k} \end{pmatrix}, \tag{4}$$

where G_k is the shear modulus of the material layer k.

The displacement state proposed by relations (1) satisfies the continuity conditions for displacements and stresses at the separating surfaces between the layers, as well as zero insurances and stresses on the outer surfaces of the bar.

The parameters that characterize the rigidity of the bar are:

$$\begin{aligned} \langle EI_1 \rangle &= 2b \sum_{k=1}^{n} E_k \left(\frac{y_k^3 - y_{k-1}^3}{3} \quad \frac{y_k^5 - y_{k-1}^5}{5} \right) \mathrm{B}_k, \\ \langle EI_2 \rangle &= 2b \sum_{k=1}^{n} E_k \, \frac{y_k^3 - y_{k-1}^3}{3}, \\ \langle GA \rangle &= 2b \sum_{k=1}^{n} G_k \left(y_k - y_{k-1} \quad y_k^3 - y_{k-1}^3 \right) \mathrm{B}_k. \end{aligned} \tag{5}$$

The parameters that characterize the inertial properties of the bar are:

$$\begin{aligned} \langle \rho A \rangle &= 2b \sum_{k=1}^{n} \rho_k \left(y_k - y_{k-1} \right), \\ \langle \rho I_1 \rangle &= 2b \sum_{k=1}^{n} \rho_k \left(\frac{y_k^3 - y_{k-1}^3}{3} \quad \frac{y_k^5 - y_{k-1}^5}{5} \right) \mathrm{B}_k, \\ \langle \rho I_2 \rangle &= 2b \sum_{k=1}^{n} \rho_k \, \frac{y_k^3 - y_{k-1}^3}{3}, \end{aligned} \tag{6}$$

where ρ_k is the layer density k.

For homogeneous thickness bar h:

$$\langle EI_1\rangle = \frac{bh^3}{15}E\,,\quad \langle EI_2\rangle = \frac{bh^3}{12}E\,,$$
$$\langle \rho I_1\rangle = \frac{bh^3}{15}\rho\,,\quad \langle \rho I_2\rangle = \frac{bh^3}{12}\rho\,, \tag{7}$$
$$\langle \rho A\rangle = \rho bh\,,\quad \langle GA\rangle = \frac{2}{3}bhG\,.$$

It is noted:

- section area: $$A = bh; \tag{8}$$
- geometric moment of inertia: $$I = \frac{bh^3}{12}; \tag{9}$$
- average density: $$\rho = \frac{\langle \rho A\rangle}{A} = \frac{2}{h}\sum_{k=1}^{n}\rho_k\left(y_k - y_{k-1}\right); \tag{10}$$
- average rotation: $$\theta = \frac{-1}{\langle EI_2\rangle}\iint_{(S)} yE(x,y)u_x(x,y,t)dS = \frac{\partial w}{\partial x} - \frac{\langle EI_1\rangle}{\langle EI_2\rangle}\alpha; \tag{11}$$
- average shear modulus: $$G = \frac{1}{A}\iint_{(S)} G(x,y)dS = \frac{2}{h}\sum_{k=1}^{n}G_k\left(y_k - y_{k-1}\right); \tag{12}$$
- average modulus of elasticity: $$E = \frac{\langle EI_2\rangle}{I}. \tag{13}$$

With these notations the shear force and the moment when the relations are calculated:

$$T = KGA\left(\frac{\partial w}{\partial x} - \theta\right), \tag{14}$$
$$M = EI\,\frac{\partial \theta}{\partial x}. \tag{15}$$

The equations of motion for the transverse vibrations of the bar are:

$$\rho A\ddot{w} - KGA\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \theta}{\partial x}\right) = p_y, \tag{16}$$

respectively

$$\rho I\left(K_1\frac{\partial \ddot{w}}{\partial x} - K_2\ddot{\theta}\right) + KGA\left(\frac{\partial w}{\partial x} - \theta\right) + EI\,\frac{\partial^2 \theta}{\partial x^2} = 0, \tag{17}$$

in which

$$K = \frac{\langle GA\rangle}{GA}\cdot\frac{\langle EI_2\rangle}{\langle EI_1\rangle}, \tag{18}$$
$$K_1 = \frac{\langle \rho I_1\rangle\langle EI_2\rangle - \langle \rho I_2\rangle\langle EI_1\rangle}{\rho I\langle EI_1\rangle}, \tag{19}$$
$$K_2 = \frac{\langle \rho I_1\rangle\langle EI_2\rangle}{\rho I\langle EI_1\rangle}, \tag{20}$$

are the coefficients that take into account the non-uniformities of the stresses in the section. In the case of the homogeneous bar

$$K = \frac{5}{6}, K_1 = 0, K_2 = 1,$$

and the equations of motion are identical to those of classical Timoshenko's bar theory.

It is considered a bar of three layers, a middle layer and two outer layers, arranged symmetrically. In current applications, the outer layers have mechanical and strength properties superior to the middle layer.

Figure 2 show variations of the three coefficients of non-uniformity depending on the ratios between the modulus of elasticity and the thickness of the layers for $\frac{G_2}{G_1} = 10$.

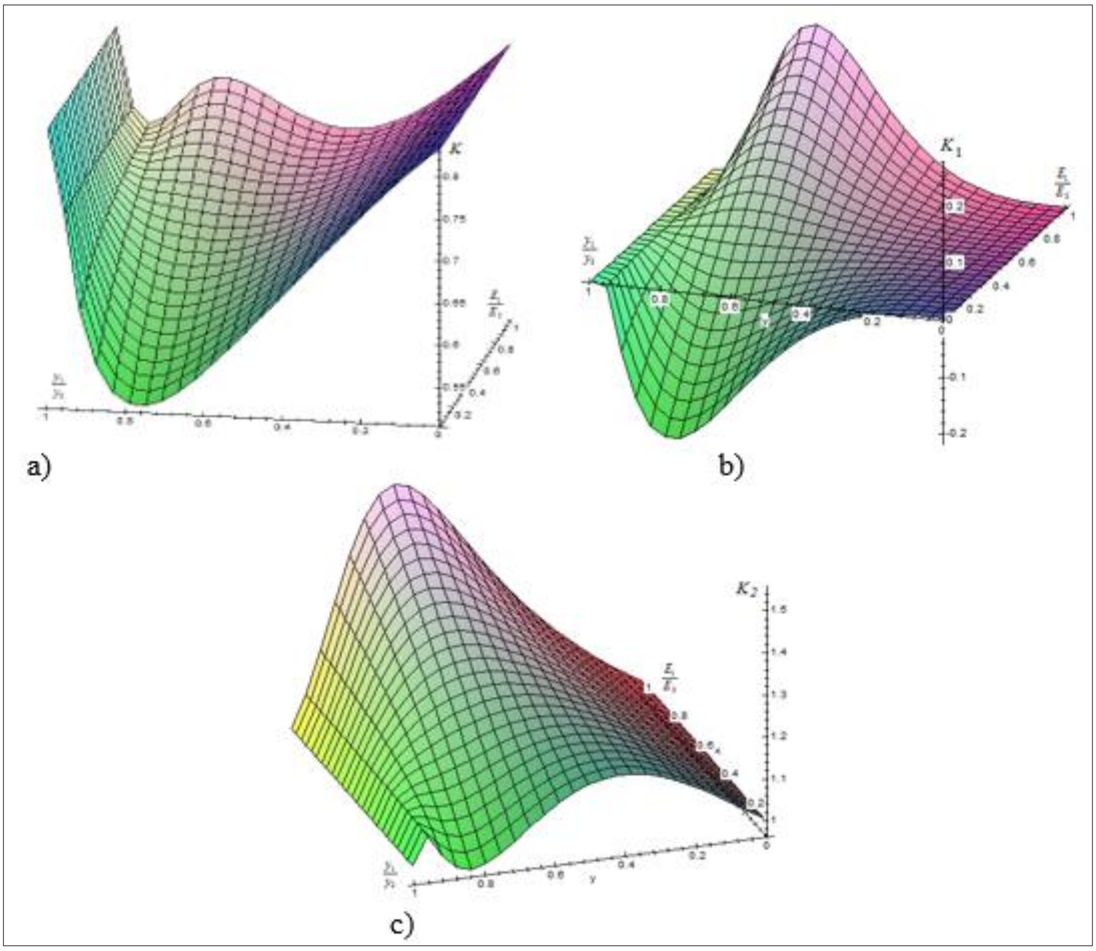


Figure 2. Variation of the unevenness coefficient: K a); K_1 b); K_2 c)

It should be noted that in the classical theory of bars Timoshenko is used a single coefficient of non-uniformity for stresses. In the presented model there are three coefficients that characterize the non-uniformities of the stresses in the bar section. If the coefficient takes into account the variations of the tangential stresses, the coefficients and appreciate the variations of the normal stresses. By customizing these coefficients, mathematical models for the vibrations of laminated composite bars presented in [29] or [30] are obtained.

The method of analysis of the mathematical model is presented in detail, for different boundary conditions in [31]. It is shown that the simplified relation can be used to calculate the equivalent stiffness $\langle EI_2 \rangle$:

$$\langle EI_2 \rangle = \frac{\langle \rho A \rangle \omega_n^2 L^4}{\beta_n^4}, \quad (21)$$

in which β_n, $n \in N^*$, depending on the boundary conditions of the bar.

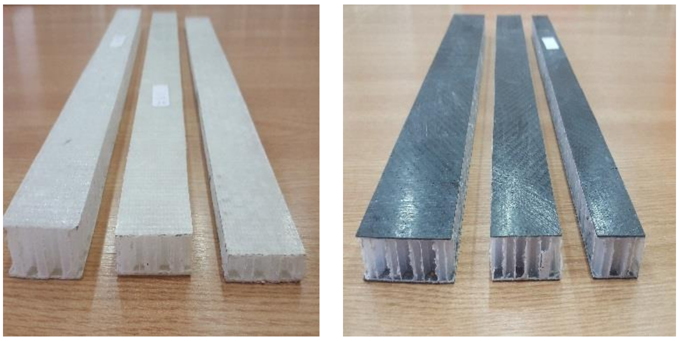
The simplified relationship is used because, in reality, all vibrations are damped. In the case of damped vibrations, there is a decrease or decrease in vibration pulses. An announced study of the damping mechanisms is presented in [32]. It is shown that the length of the bar has a very large influence on the damping, depending on the type of damping, the damping factor being constant, inversely proportional to the square of the length of the bar or inversely proportional to the fourth power of the length of the bar.

3. Experimental part

Polypropylene honeycomb core boards with a thickness of 10 mm, 15 mm and 20 mm are made, on the faces of which one or two layers of carbon fiber and fiberglass fabric have been added. The average thickness of the reinforced layers with carbon fiber fabric was 0.3 mm, and the average thickness of the reinforced layers with fiberglass fabric was 0.45 mm. Resoltech 1050 epoxy resin and the corresponding Resoltech 1055 hardener were used to make the sandwich composite materials. To make the outer faces of the sandwich composite, the carbon fiber and glass fiber fabrics were impregnated with epoxy resin and they were spread out on a flat surface. A polypropylene core was placed over the fiber layers and weights were added to ensure a pressure of $5N/cm^2$. The adhesion between the added layers and the core was achieved due to the fact that, from the production phase, the honeycombs had added polypropylene fabrics on the outer surfaces. The epoxy resin penetrated through the fabric into the honeycomb cells, and by hardening it ensured the stiffening of the sandwich composite. The technical data of the epoxy resin (Resoltech 1050 / Resoltech 1055) can be found on the manufacturer's website ([Resoltech 1050]).

From each plate bars are cut with a length of 400 mm and widths of 30 mm, 45 mm and 60 mm respectively.

Figure 3a shows bars with outer layers reinforced with fiberglass, 4.5 cm wide, with a thickness of the middle layer of honeycomb of 20 mm, 15 mm and 10 mm, respectively. Figure 3b shows bars with outer layers reinforced with carbon fiber, with a thickness of the middle layer of honeycomb of 20 mm and widths of 3 cm, 4.5 cm and 6 cm.



(a) (b)

Figure 3. Medium-layer polypropylene honeycomb bars and outer layers with fiberglass (a) and carbon fiber fabric (b)

For these sets of specimens, the eigenfrequency and the damping coefficient, corresponding to the first eigen vibration mode are experimentally determined.

The measuring equipment used was:

- accelerometer with a sensitivity of 0.04;
- SPIDER 8 data acquisition system;

- NEXUS 2692-A-0I4 signal conditioner connected to SPIDER 8 system.

The bars were embedded in a section, then left free lengths: L = 200; 230; 260; 290; 320; 350 [mm].

The measuring point (the point where the accelerometer was placed) is according to the diagram in Figure 4. Each measurement for each point was performed twice.

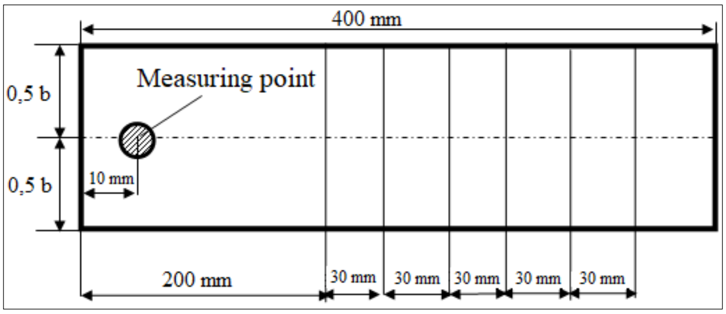


Figure 4. Position of the measuring point where the accelerometer was located

Figure 5 shows an experimental record, and Figure 6 shows the frequency and damping factor determination for the 3 cm wide bar, 1 cm thick honeycomb, a layer of carbon fiber fabric, and the console length of 200 mm.

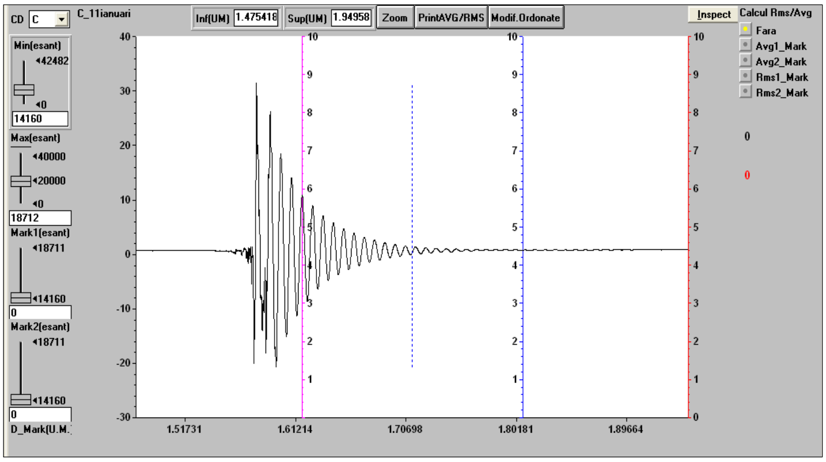


Figure 5. Experimental recording for the 3 cm wide bar, with
a 1 cm thick honeycomb, a layer of carbon fiber fabric and a console length of 200 mm

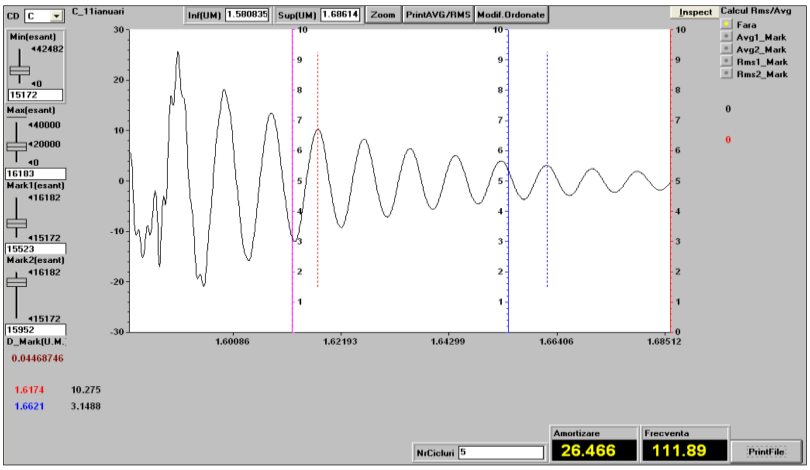


Figure 6. Determination of the damping factor for the 3 cm wide bar, with a 1 cm thick honeycomb, a layer of carbon fiber fabric and a console length of 200 mm

Figures 7-8 show the damping factor and the vibration frequency of the bar with carbon fiber fabric faces. The values introduce arithmetic means of the results obtained from two experimental determinations.

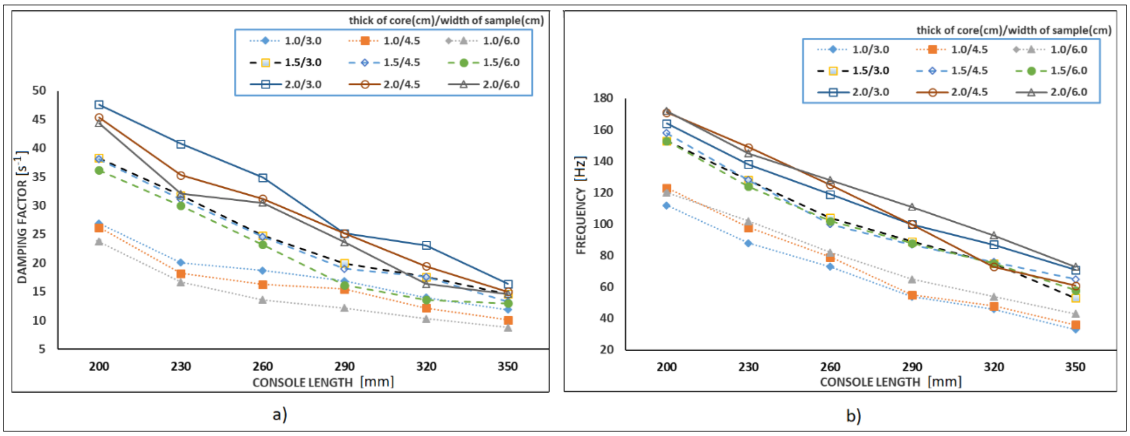


Figure 7. Damping factor (a) and frequency (b) of vibrations of single-layer face bars made of carbon fiber fabric

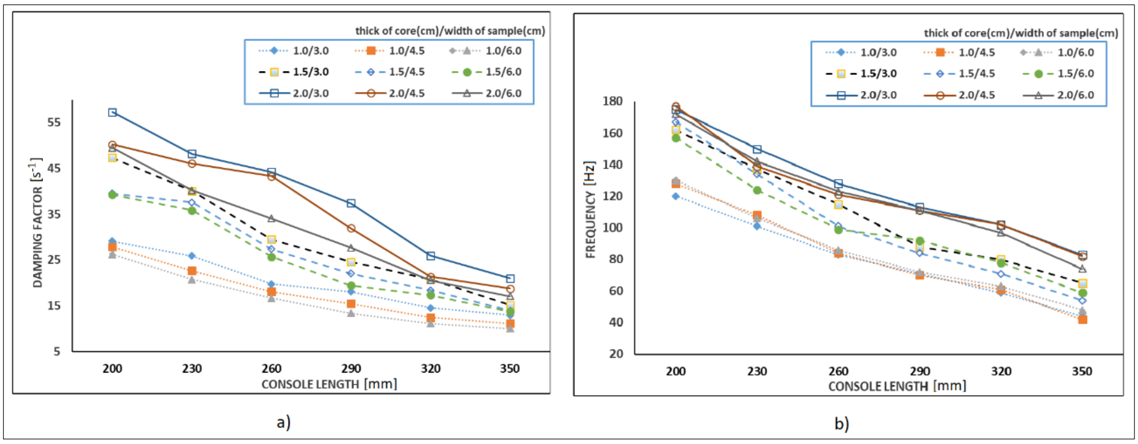


Figure 8. Damping factor (a) and frequency (b) of vibrations of two-layer face bars made of carbon fiber fabric

It is observed that the dimensions of the core influence the vibration behavior of the studied composites. The damping factor increases with the thickness of the honeycomb used as the core. This can be explained by the fact that the energy dissipates in a larger amount of material and therefore increases the ability to absorb vibrations. The same increase is found in the case of vibration frequency but this can be explained by the increased rigidity of the bar sections.

Figures 9-10 show the damping factor and the vibration frequency of the bar for the bars with fiberglass fabric faces. The values introduce arithmetic means of the results obtained from two experimental determinations.

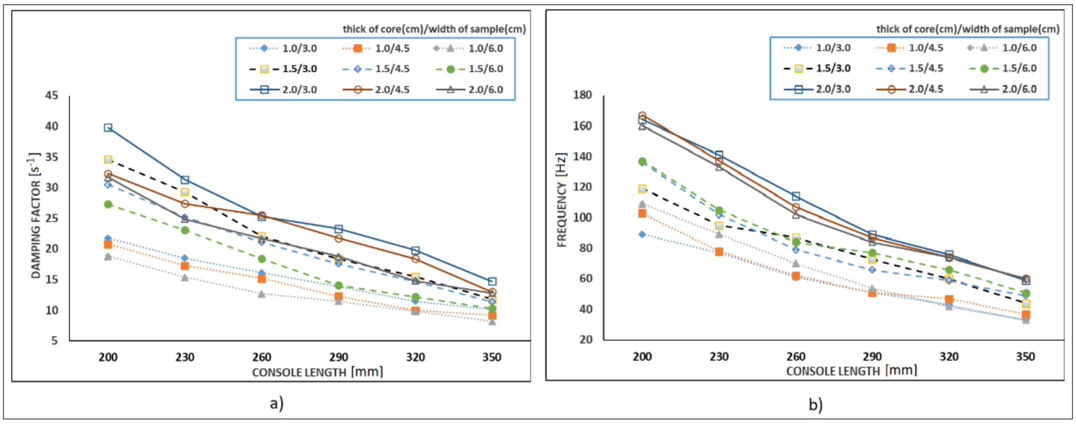


Figure 9. Damping factor (a) and frequency (b) of vibrations of single-layer face bars of fiberglass fabric

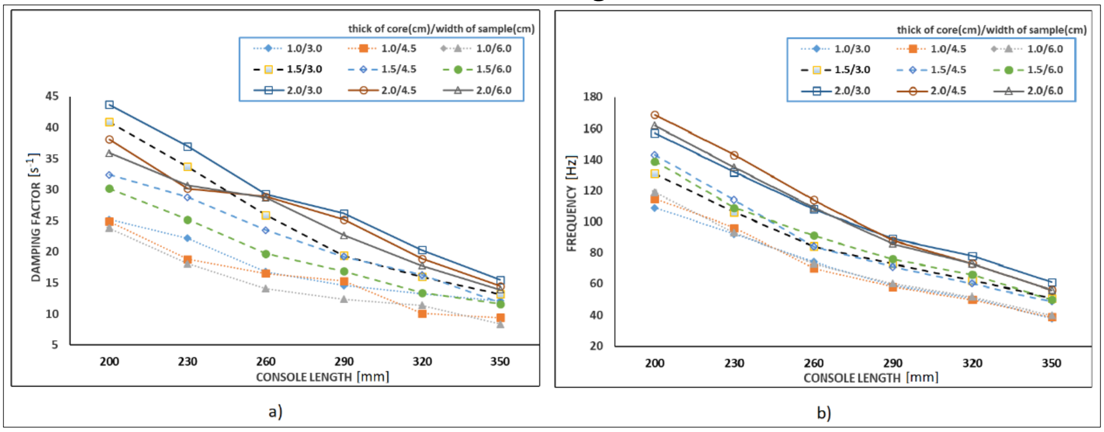


Figure 10. Damping factor (a) and frequency (b) of vibrations of two-layer face bars of fiberglass fabric

And for fiberglass fabric girls sandwich bars, a vibration behavior similar to carbon fiber fabric girls' composites is obtained. As the values for the damping factor and for the frequency have lower values, it results that the layers of glass fiber fabric have damping and stiffness properties inferior to the layers of carbon fiber fabric.

The experimentally obtained frequencies were used to calculate the equivalent stiffness and the average modulus of elasticity at all studied bars. Table 1 and Table 2 show the results obtained.

Table 1. Equivalent stiffness and average modulus of elasticity for bars with outer layers reinforced with carbon fiber fabric

Thick Honey Comb [mm]	Test-width [mm]	1 layer			2 layers		
		Specific linear mass [g/m]	Rigidity $<EI_2>$ [Nm²]	Equivalent modulus of elasticity [*MPa*]	Specific linear mass [g/m]	Rigidity $<EI_2>$ [Nm²]	Equivalent modulus of elasticity [*MPa*]
10	30	70	5,772	1884	100	10,079	2721
	45	105	8,543	1812	150	15,280	2750
	60	140	11,966	1953	200	21,055	2842
15	30	80	12,461	1287	115	19,033	1725
	45	115	18,687	1288	165	26,412	1596
	60	150	23,716	1226	220	35,452	1607
20	30	95	20,218	912	130	33,080	1349
	45	145	29,537	888	190	47,307	1287
	60	190	42,828	965	245	58,220	1208

Table 2. Equivalent stiffness and modulus of average elasticity for bars with outer layers reinforced with fiberglass fabric

Thick Honey comb [mm]	Test-width [mm]	1 layer			2 layers		
		Specific linear mass [g/m]	Rigidity <EI_2> [Nm^2]	Equivalent modulus of elasticity [MPa]	Specific linear mass [g/m]	Rigidity <EI_2> [Nm^2]	Equivalent modulus of elasticity [MPa]
10	30	95	5,512	1702	140	10,668	2597
	45	140	8,354	1720	210	16,056	2606
	60	185	11,693	1805	275	21,611	2630
15	30	105	10,666	1061	155	17,692	1492
	45	155	15,786	1047	230	26,052	1465
	60	205	23,305	1159	300	36,367	1534
20	30	120	21,104	912	170	29,029	1105
	45	175	30,063	865	250	43,197	1097
	60	230	38,414	830	325	55,470	1037

It is observed that for the bars obtained from the same mesh the modulus of elasticity is very close. Table 3 shows the average modulus of elasticity for all studied materials.

Table 3. Average modulus of elasticity (in *MPa*)

Thickness comb	Carbon fiber fabric		Glass fiber fabric	
	1 layer	2 layers	1 layer	2 layers
10 mm	1883	2771	1742	2611
15 mm	1267	1642	1089	1497
20 mm	922	1281	869	1080

4. Conclusions

The mathematical model proposed for the study of the vibrations of multilayer composite bars makes a generalization for the Timoshenko model, taking into account both the non-uniformities in the section of tangential stresses and the non-uniformities of normal stresses. Tangential stresses and normal stresses in all layers are taken into account, with all conditions of continuity on the separation surfaces between the layers being met, both for the stress tensor components and for the strain tensor components. It is also taken into account that the stresses on the outer surfaces are zero. If in Timoshenko's theory a coefficient is introduced that counts non-uniformities in the section of tangential stresses, in the proposed mathematical model three such coefficients are introduced which also count for non-uniformities of normal stresses. The analysis of the graphical representations for the three coefficients of non-uniformity and of the ratios between the mass and stiffness characteristics of the bar, in the case of the sandwich bar, shows the existence of significant variations both depending on the mass and elastic properties of the component layers. relatively ale. them. If the mass and elastic properties of the layer materials are very close, the values of the non-uniformity coefficients are those for homogeneous materials. The proposed model, through the variation in the section of stresses and strains, can be introduced in PSDBT (parabolic shear deformation beam theory).

From the analysis of Tables 1-3 it can be concluded that:

- the equivalent rigidity increases with the number of fabric layers used, the increase being about 90% for the 10 mm honeycomb bars, then decreasing to about 40% for the 20 mm honeycomb bars;
- the equivalent rigidity is proportional to the width of the bar; the difference may also occur because when cutting the bars, the honeycomb sections may differ from one bar to another;
- the average modulus of elasticity decreases with the thickness of the middle layer, which can be explained by the fact that approximately the same amount of fabric is inserted into a smaller section at the 10 mm honeycomb bars, which increases at the 15 mm honeycomb core bars, respectively 20 mm;
- the average modulus of elasticity is higher for bars with outer layers reinforced with carbon fiber fabric than for those reinforced with glass fibers, which can be explained by the fact that carbon fibers have a higher modulus of elasticity than glass fibers.

From the analysis of Figures 7-10 it can be concluded that:

- the damping factor increases with the thickness of the honeycomb that constitutes the middle layer;
- the damping factor is higher for bars with two outer layers of fabric than for bars with a single outer layer of fabric;
- the damping factor is higher for bars with outer layers of carbon fiber fabric than for bars with outer layers of fiberglass fabric.

The values of the damping coefficient and those of the vibration frequency of the bars made of sandwich composite materials with polypropylene honeycomb core (with various thicknesses) and faces of fiberglass fabric, respectively carbon fiber fabric, show that these types of materials can be used for the floors of means of transport (airplanes, trucks, etc.), in construction, as well as in the medical field. For example, X-ray scanning systems equipped with carbon fiber face composite top plates can achieve high quality scans compared to traditional systems. Instead, sandwich composites with fiberglass fabric faces can be used as partitions for radiation protection for healthcare workers working with X-ray sources (such as spectrometers and computed tomography (CT) scanners).

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