Experimental Researches Concerning the Properties of Composite Materials with Random Distribution of Reinforcement

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In this paper we present the result of the experiments performed to obtain the characteristics curves for five composite materials with random distribution of reinforcement. From these curves we determined the elasticity modulus and the resistance to fracture. Using the modal identification method we determined the first eight eigenmodes for two bars from composite materials (Bar 1 – phenolic fireproof resin reinforced with fiberglass; Bar 2 – ortophtalic polyesteric resin reinforced with fiberglass), embedded at one end and free at the other. We determined the eigenpulsations for the modes considered and we used the first four modes for the calculus of elasticity modulus. The results obtained by traction testing compared with the one from modal identification method certify the utilization of modal analysis in the determination of properties of composite structures.

Keywords: composite materials, elasticity modulus, modal identification method

The composite materials enable to obtain a great diversity of mechanical properties. This fact makes difficult the determination of mechanicalcharacteristics depending on the proportion of composite components. The existing theories point out the composites like homogeneous, generally anisotropic where the material constants are obtained function of the properties of the constituents. Usually good results are obtained in static problems, but serious deficiencies appear concerning the vibrations, especially due to attenuation which was observed in the case of composite materials.

A frequent used theory is the theory of the blends, based on the elementary similitude with the blends of gases, in which the constituents coexist, each exercising the own partial pressure. With the observance of composite structure, the constituents are presumed heteronymous in space, each having individual deformations. The laws of the blends can be easily formulated, but the principal problem of the application of the theory of the blends in the case of composite blends is the analytic specification of interactions of constituents and of the constituent equations for the blend, being known the geometrical distribution and the constituent equations for every individual component. The complexity of this problem can be presented considering the case of fibers arrangement in an unidirectional composite

If the fibers are arranged regularly, it is possible the identification of an elementary cell that is repeated in section. But, in the routine, at the realization of composite materials, the fibers are randomly distributed, some of them being included entirely in the matrix, as the others keep contact between them. From the analytic point of view, the real solution is found out between the solution for the case when the fibers are isolated among them and the solution in the case when they are in contact.

The results obtained harmonize well with the reality only in the Young's modulus case along the fibers and in Poisson's coefficient case longitudinal, the rest being able to be used only to find out the order of size (measure, proportion) of elastic coefficients.

A series of simple relations for Young modulus and Poisson coefficient along the fibers which according to those given by blends theory are proposed starting from the results theoretically obtained in paper [1].

For the others elastic coefficients it is also suggested one relation, but which has the disadvantage to depend on a parameter that characterizes the interaction fibers-matrix, the geometry of the fibers, their arrangement, but this parameter must be determined in an empiric manner.

For some composite reinforced with fibers, the existence of a nonlinear relation among tension–deformation had been shown [2].

For instance, at the composites epoxidic boron-resin or epoxidic bleak lead-resin, the nonlinear behaviour is due to the substance of the matrix that affects basically the slip module, while the relations tension-deformation on the fibers direction and also on the transverse direction remain almost linear.

Methods of analysis of nonlinear relation of constituents were suggested as well [3-6].

Also, some types of load were considered in the case of plates with nonlinear properties [7-8].

The answer of the reinforced plates with fibers, at the dynamic charge was determined in [9], while in [10] led a research on the nonlinearity physical influence on the dynamic behaviour of the composite plates.

The dynamic behaviour for composite bars subjected to the solicitation of shock types were researched [11-13].

In [14] were obtained theoretical results and experimental determinations. Using a matrix method were determined the main elastic characteristics of composite materials and their variation depending on the volumetric proportion of reinforcement.

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The influences of damage of material on the vibrations of composite bars with thin layers were determined [16-17].

The characteristics curves

Due to the complexity of the problem and to the big number on the parameters on which the properties of composite materials, depend they should be experimentally verified it if necessary to be experimentally certified.

Very good results are obtained in the case of a testing to stretch. Besides the tension-deformation dependency it is also obtained the elasticity modulus, resistance to fracture and the breaking tension.

We achieved the traction testing for five samples as follows:

-Sample 1 built from phenolic fireproof resin reinforced with fiberglass;

-Sample 2 built from polyesteric ortophtalic resin reinforced with fiberglass;

-Sample 3 built from propylene – sulphonates;

-Sample 4 built from polyesteric fireproof resin reinforced with fiberglass;

-Sample 5 built from acrylic polymer.

In figure 1. (Sample 1), figure 2 (Sample 2), figure 3 (Sample 3), figure 4 (Sample 4) and figure 5 (Sample 5) we present the characteristic curves for the composite materials of each plate.

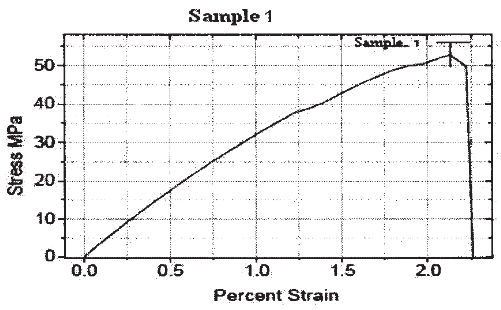

Fig. 1. Characteristic curve (Sample 1)

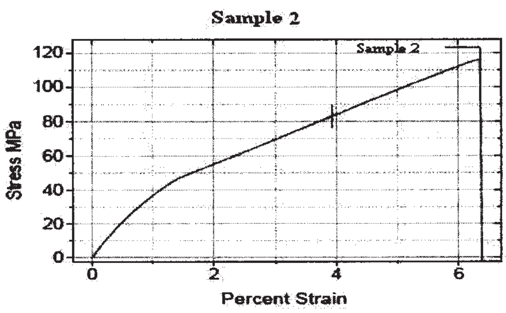

Fig. 2. Characteristic curve (Sample 2)

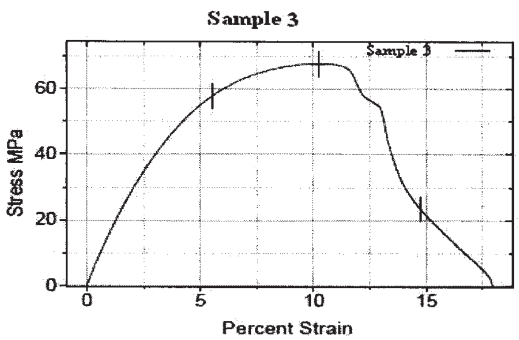

Fig. 3. Characteristic curve (Sample 3)

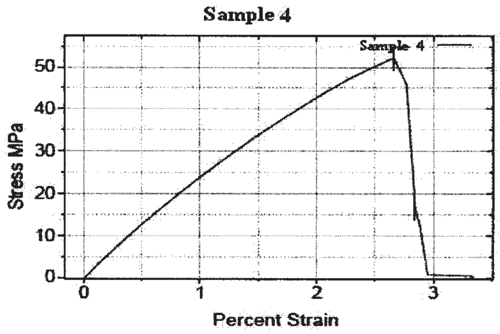

Fig. 4. Characteristic curve (Sample 4)

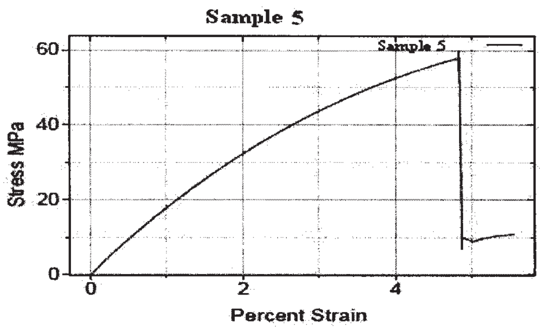

Fig. 5. Characteristic curve (Sample 5)

In table 1 are presented the elasticity modulus and the resistance to fracture for the five samples tested.

In the case of Sample 3 were introduced the elongation for the maximum value of tension because appeared a flowing phenomenon, the elongation in the moment of fracture being 17 %.

Theoretical background regarding the experimental modal identification

Any mechanical system can be modeled by “*n*” concentrated mass points “m_k”, joint by elastic elements with “k_k” stiffness and elements with “c_k” damping. For this damped system with „“*n*” degrees of freedom, subjected to an external excitation system {Q(t)},the motion equations are given by the following relation:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{Q(t)\}, \tag{1}$$

where

- [M], [C], [K]- the mass, damping and stiffness matrices;

- $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$, $\{x(t)\}$- the acceleration, velocity and displacement vectors;

- {Q(t)}- generalized forces vector.

The system response to an external excitation is presented as a sum of “*n*” modal contributions due to each separate degree of freedom:

$$\{X(\omega)\}=\sum_{k=1}^{N}\left[\frac{\{\psi^k\}\cdot\{\psi^k\}^T\cdot\{Q(\omega)\}}{a_k(-\mu_k+i(\omega-\nu_k))}+\frac{\{\overline{\psi^k}\}\cdot\{\overline{\psi^k}\}^T\cdot\{Q(\omega)\}}{\overline{a}_k(-\mu_k+i(\omega+\nu_k))}\right], \tag{2}$$

where: X(ω) - Fourier Transform of displacement;

Table 1

Sample	1	2	3	4	5
Elasticity modulus (Mpa)	3660	4581	1710	2815	2066
Resistance to fracture (Mpa)	53	116	68	52	58
Elongation to fracture (%)	2,2	6,4	11	2,7	4,8

$\{\psi^k\}$ and $\{\overline{\psi}^k\}$ - the “k” order eigenvector and his complex conjugate;

μ_k - the “k” order damping ratio;

ν_k - the “k” order damped natural frequency;

a_k and $\overline{a}_k$ - norming constants of eigenvector;

ω - external excitation frequency.

In practical applications the modal vectors are replaced by two modal constants U_{ij}^k and V_{ij}^k defined by:

$$\frac{\psi_i^k \cdot \psi_j^k}{a_k} = U_{ij}^k + i \cdot V_{ij}^k \quad \text{and} \quad \frac{\overline{\psi}_i^k \cdot \overline{\psi}_j^k}{\overline{a}_k} = U_{ij}^k - i \cdot V_{ij}^k . \tag{3}$$

Using these notations we can determine the system admittance, $\alpha_{ij}(\omega)$ defined as the rapport between the displacement response and the force excitation:

$$\alpha_{ij}(\omega) = \sum_{k=1}^{n} \left[\frac{U_{ij}^k + i \cdot V_{ij}^k}{-\mu_k + i \cdot (\omega - \nu_k)} + \frac{U_{ij}^k - i \cdot V_{ij}^k}{-\mu_k + i \cdot (\omega + \nu_k)} \right] \tag{4}$$

In the approximations made during the conception of the mathematical model, it was used the concept of discrete system with “*n*” liberty degrees, having its mass concentrated in “*n*” material points. For a precise approximation of the real system by a discrete system “*n*”must have a high value ($n \to \infty$). This is not possible because of experimental and processing techniques and also because of the time needed for data processing. In applications the frequencies domain is limited to a reasonable width determined by the major resonances of the analyzed equipment and the frequency domain of application target. In these conditions the sum from (4) is reduced to several components, noted in the following with “*n*” too. The contributions of inferior and superior modes are included in two correction factors known as “inferior modal admittance”, $-1 / M'_{ij}\omega^2$.(for inferior modes) and “residual flexibility”,S'_{ij} (for superior modes). The system admittance will be written as:

$$\alpha_{ij}(\omega) = \frac{-1}{M_{ij}' \cdot \omega^2} + \sum_{k=1}^{n} \left(\frac{U_{ij}^k + i \cdot V_{ij}^k}{-\mu_k + i \cdot (\omega - \nu_k)} + \frac{U_{ij}^k - i \cdot V_{ij}^k}{-\mu_k + i \cdot (\omega + \nu_k)} \right) + S_{ij}' . \tag{5}$$

Parameters modal experimental identification

In the following is presented the experiment for modal identification of two plates: the first plate from phenolic fireproof resin reinforced with fiberglass (Sample 1) having the dimensions 600mm . 100 mm . 4.6 mm and the second plate from ortophtalic polyesteric resin reinforced with fiberglass (Sample 2) having the dimensions 600 mm . 100 mm . 4.4 mm.

The plates are rigidly fixed in a press with a mass of 80 Kg. The experimental montage is presented in figure 6.

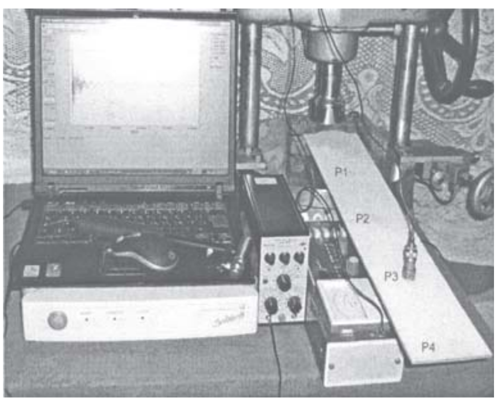

Fig.6. Experimental montage

Apparatus measuring system

Accelerometer B&K type 4391 (m=30g; Exciter B&K type 8202; Conditioning amplifier B&K type 2626; Charge amplifier type M1300; Digital measuring system Spider 8.

Experiment

The plates were divided in four points equally distributedP1,....,P4. The accelerometer was successively mounted in points P1,...,P4and for each measuring point the plates were successively excited in points P1,....,P4.

For each excitation conditions was measured the excitation force and the plate acceleration response. Fig. 7 (Sample 1) and figure 8 (Sample 2) present the experimental data corresponding to excitation in point and measuring in point P4.

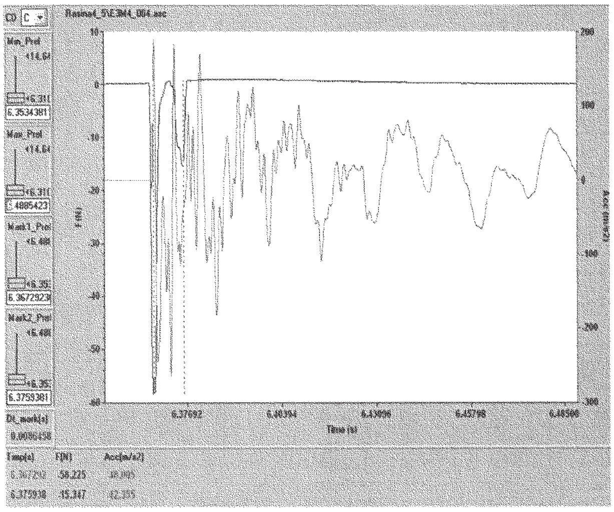

Fig. 7. Time recorded characteristics (Sample 1)

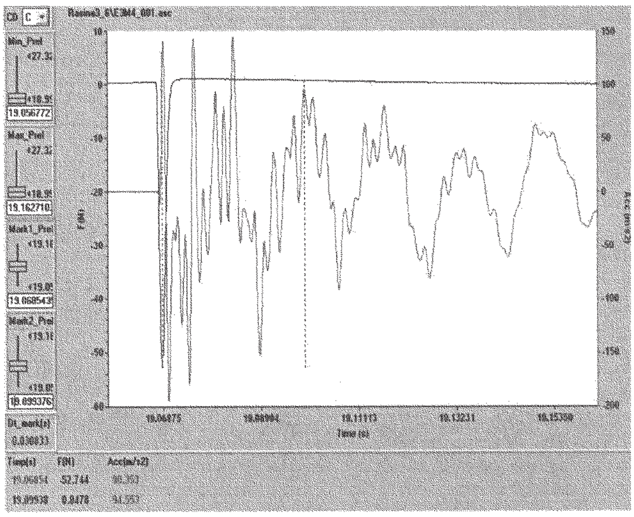

Fig. 8. Time recorded characteristics (Sample 2)

Modal identification

It was made under Test Point software, a package of programs for modal identification and for estimation of the structural response to external excitation distributed into the structure or concentrated in distinct points. The modal identification is made by the following next steps:

1. Determination of the frequency response functions. In the range of 0 … 500 Hz frequencies, the plates have eight eigenmodes. figure 9 (Sample 1) and figure 10 (Sample 2) present the frequency response function (FRF) in Cartesian coordinates and figure 11 (Sample 1) and figure 12 (Sample 2) in polar coordinates,

corresponding to excitation in point $P3$ and measuring in point P4.

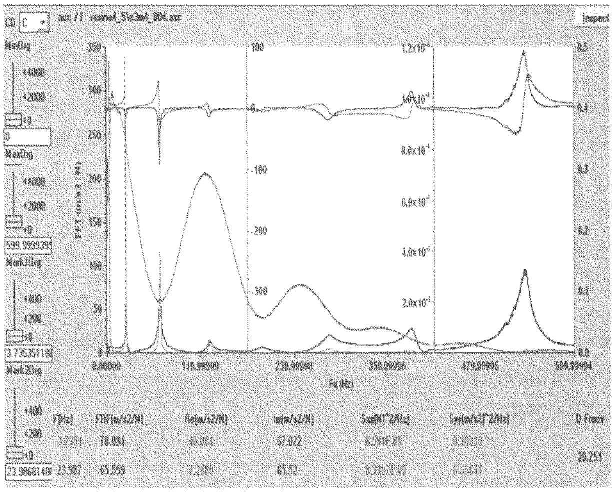

Fig. 9. FRF in Cartesian coordinates (Sample 1)

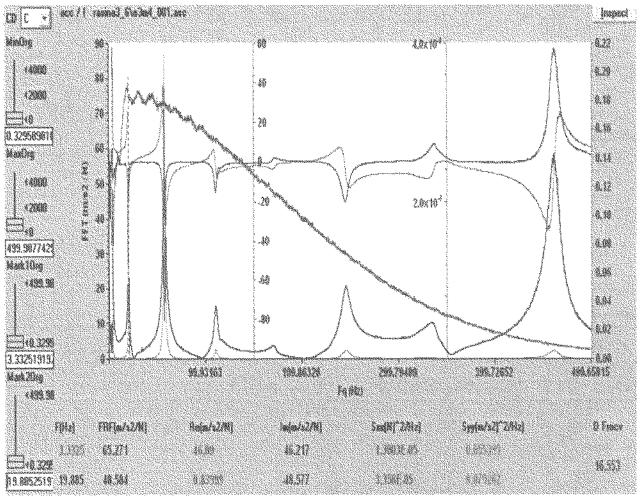

Fig. 10. FRF in Cartesian coordinates (Sample 2)

2. The approximate localization of the resonances and determination in initial approximation of the modal parameters μ_k and ν_k, k=1,2,...,n.

3. The first stage identification of modal parameters

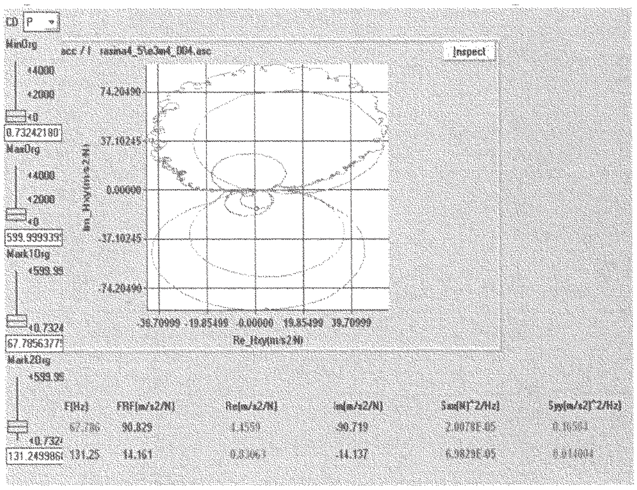

Fig. 11. FRF in polar coordinates (Sample 1)

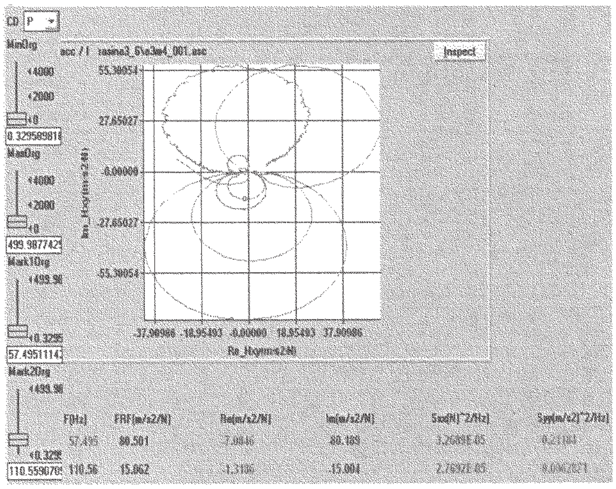

Fig. 12. FRF in polar coordinates (Sample 2)

$$\mu_k\,,\ \nu_k\,,\ U_{ij}^{\,k}\,,\ V_{ij}^{\,k}\,,\ -\frac{1}{M_{ij}^{'}}\,,\ S_{ij}^{'}\ ,$$

on limited frequency domains. The identification is made using linear procedures, determining those modal parameters which inserted in relation (5) generate theoretical characteristics which approximate with minimal error the experimentally determined FRF.

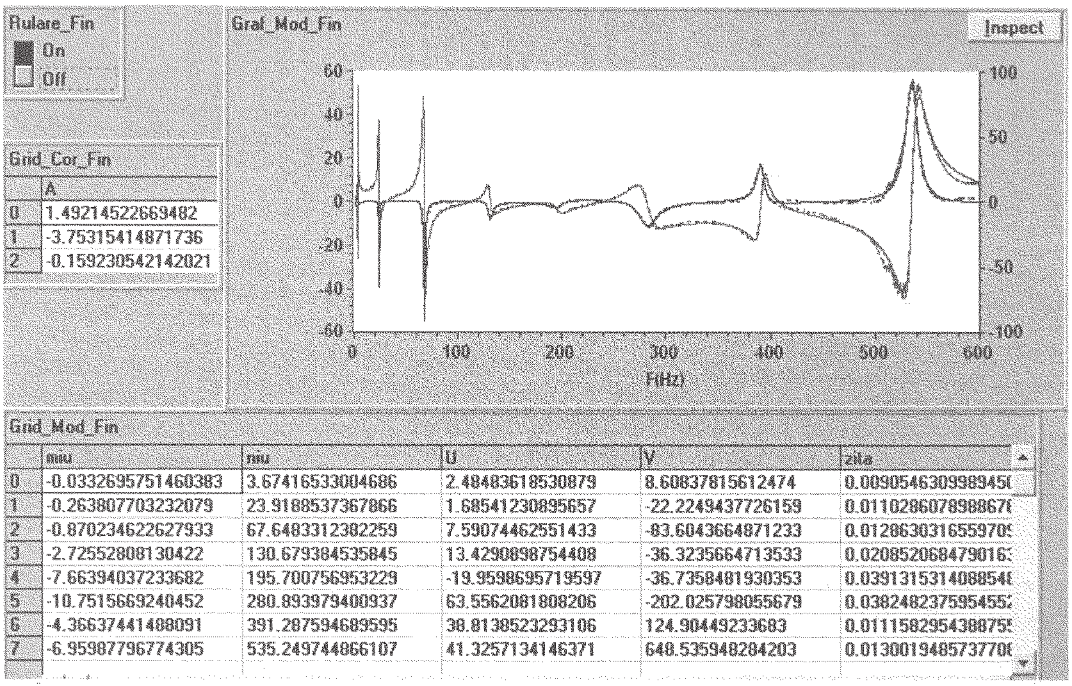

Fig. 13. Final panel with modal parameters and theoretic and experimental characteristics (Sample 1)

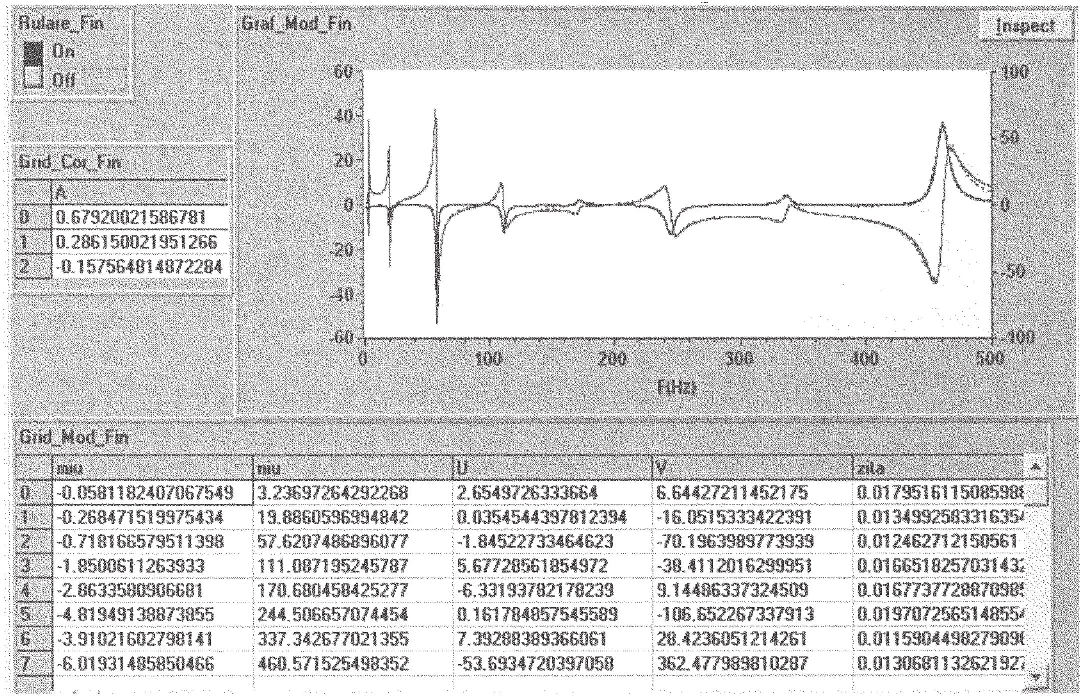

Fig. 14. Final panel with modal parameters and theoretic and experimental characteristics (Sample 2)

4. The final identification of the modal parameters

$$\mu_k, \nu_k, U_{ij}^k, V_{ij}^k, -\frac{1}{M_{ij}^{'}}, S_{ij}^{'}, \quad k = 1,2,...,n$$

over entire frequency range. The identification is made using nonlinear procedure of recursive approximation, determining those modal parameters which inserted in relation (5) generate theoretical characteristics which approximate with minimal error the experimentally determined frequency response functions.

The figure 13 (Sample 1) and figure 14 (Sample 2) presents the final panel of identification module in which are represented in Cartesian coordinates the real and imaginary parts of theoretic and experimental frequency response functions. In the second part of the page it is a table containing the modal parameters for all eigenmodes.

It can be observed that there are very little differences between the experimental and theoretical characteristics.

The modal parameters can be used for analysis of structural modifications and for assessment of structure response to given excitations concentrated in some distinct points or distributed into the structure.

The utilization of modal analysis to determine the elasticity modulus.

The transversal vibrations equation of bars is:

$$EI \cdot \frac{\partial^4 W}{\partial x^4} + \rho A \cdot \frac{\partial^2 W}{\partial t^2} = f(x,t), \tag{6}$$

where:

W(x,t) is the transversal movement of medium fiber of the bar;

E - is the elasticity modulus of the bar material;

I - the moment of axial inertia of the bar section;

ρ - the density of the bar material;

f(x,t) - extrinsically force which reacts on the bar.

If the bar is embedded at one end and free at the other the eigenfunctions have the form [15]:

$$W_r(x) = C\left[\frac{ch(\beta_r lx) - \cos(\beta_r lx)}{2} - \frac{ch\beta_r + \cos\beta_r}{sh\beta_r + \sin\beta_r} \cdot \frac{sh(\beta_r lx) - \sin(\beta_r lx)}{2}\right], r \in N^*. \tag{7}$$

The eigenfrequencies are[15]:

$$\upsilon_r = \frac{\beta_r^2}{2\pi l^2}\sqrt{\frac{EI}{\rho A}}, \tag{8}$$

where:

l is the bar length ;

β_r - are the roots of equation:

$$ch\beta \cdot \cos\beta + 1 = 0. \tag{9}$$

We have: β =1.875; β =4.694; β =7.855; β =10.996; β =14.137; β =17.279; β =20.420; β =23,562

Therefore, identifying the eigenfrequencies we can determine the elasticity modulus with the relation:

$$E = 4\pi^2 l^2 \frac{\rho A}{I}\left(\frac{\upsilon_r}{\beta_r^2}\right)^2. \tag{10}$$

The characteristic bars on which we make the measurements are presented in the table 2.

Table 2

Bar	$l\,(m)$	$b\,(m)$	$h\,(mm)$	$\rho\,(Kg/m^3)$	$A\,(m^2)$	$I\,(m^4)$
1	0,6	0,1	4,6	1130	$0{,}46 \cdot 10^{-3}$	$811 \cdot 10^{-12}$
2	0,6	0,1	4,4	1700	$0{,}44 \cdot 10^{-3}$	$710 \cdot 10^{-12}$

Table 3

Mode		Mode 1	Mode 2	Mode 3	Mode 4
Frequency (Hz)	Bar 1	3,674	23,918	67,648	130,679
	Bar 2	3,237	19,886	57,621	111,087
$\frac{\upsilon_r}{\beta_r^2}$	Bar 1	1,044	1,085	1,096	1,081
	Bar 2	0,921	0,903	0,934	0,919
Elasticity modulus (MPa)	Bar 1	3574	3860	3939	3832
	Bar 2	4572	4395	4702	4552

For the determination of elasticity modulus we use the first four modes of vibration. The results are presented in table 3.

If we make the medium for Bar 1 we obtain E=3801 MPa and for Bar 2, E=4555 MPa.

Conclusions

In the case of composite materials with random distribution of reinforcement it is difficult to establish the calculus relations for the elastic and strength properties. This happened because it can't be established exactly the volumetric distribution of reinforcement and the way in which this takes over the external stresses.

The determination of composite material properties using the characteristic curves has the advantage that information concerning the elasticity modulus, strength to fracture and the fracture elongation can be obtained. From figures 1 – 5 we notice that the elastic and strength properties are closer to those of the matrix. This thing is explicable by the fact that the number of fibers oriented on the stress direction is much smaller than the total number of fibers. Consequently, the matrix takes the efforts in a greater percentage than in the case of the one-way composites.

In the case of the propilen-sulphona sample we can notice that the phenomenon of flowing appears, the substance having a ductile behaviour. In case of samples armed with glass fibers the breakage happens suddenly, it has a fragile character. It is explained by the fact that the breakage of the composite material happens in the moment when the fibers give up.

The modal identification method has the advantage that it is non-destructive and can be used in the case of complex systems built from composite materials. But it can't be used to determine the mechanical parameters characterizing the breakage. For the determination of elasticity modulus were used just the first four modes of vibration because if we take in the calculus any following modes errors appear. The superior modes undervaluate the elasticity modulus. This thing can be explained by the fact that the internal frictions and the internal dumping has effect on the modes of the superior vibrations.

The error given by the modal identification method relative to the method of stretching test for the sample 1 is 3.85% and for sample 2 is 0.57%.Therefore, the modal identification method gives very good results in finding the elasticity modulus.

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