

Transport Phenomenon Simulation for Linear Polymers through Nanometer Pores

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In this paper the linear polymers transport through a nanopore is simulated. The major effects of superimposed electric field and the monomers number on the polymer translocation have been proved. The polymer escape time is inversely proportional with the electric field's intensity and directly proportional with the polymer's length.

Keywords: transport phenomena, Langevin equation, nanometer pore, translocation time

The polymer transport process through a membrane with nanometer pores has been intensively studied [1, 2, 3]. Naturally, the sources which trigger the translocation activity can be a chemical potential difference, the selective adsorption on one side of the membrane, or an external electric field [4].

Brownian dynamics simulation is the most used molecular simulation method for the biopolymers transport processes simulation. The displacement mechanism of particles which follow Brownian motion is described by Langevin equation [5].

The general dynamics of each monomer results from the random bombardment of solvent molecules [3, 5]. In this conception the monomer motion is a Brownian motion, evidently. According to the announced scenario, any polymer molecule contains N monomers, each of size a , being virtually forced to move from a *CIS* zone to a *TRANS* zone through a pore of nanometer dimensions.

Theoretical part

a) The time integration method of Langevin equation

For easily understandable reasons, but especially due to the fact that it is computationally less demanding than ordinary Newtonian Molecular Dynamics (MD), the Langevin equation is the most used model in the macromolecular simulation problems. A classical representation of Langevin differential equation [6, 7] is the following expression

$$\begin{cases} dx = v dt \\ M dv = F(x) dt - k_B T D^{-1} v dt + 2^{1/2} k_B T D^{-1/2} dW(t) \end{cases} \quad (1)$$

where x is the displacement vector, v is the velocity vector, t is the time, M is a diagonal matrix of masses, $F(x)$ is the collective force vector, D is a constant diagonal diffusion tensor, k_B is the Boltzmann constant, T is absolute temperature and $W(t)$, $t > 0$, is a collection of independent standard Wiener processes.

A Wiener process $W(t)$, $t > 0$, is a one-parameter family of Gaussian random variables with expectations zero and covariances $E(W(s)W(t)) = \min\{s, t\}$. Because the $W(t)$ are all Gaussian, this information suffices to determine joint probabilities. Alternatively, $W(t)$ may be viewed as a "random" continuous function with $W(0) = 0$.

In the special situation when the friction tensor $k_B T D^{-1} = \gamma M$ for some scalar $\gamma \geq 0$, the Langevin equation (1) becomes

$$\begin{cases} dx = v dt \\ dv = M^{-1} F(x) dt - \gamma v dt + \sqrt{2\gamma k_B T} M^{-1/2} dW(t) \end{cases} \quad (2)$$

For the case of a diagonal diffusion tensor, the Langevin equation can be solved with the help of several known procedures, such as the van Gunsteren - Berendsen (vGB82) [8], the Brooks-Brunger-Karplus (BBK) [9] algorithms, and the "Langevin impulse" (LI) integrator [10].

Nevertheless we will not use the classical approaches presented before. From several reasons [11], the time integration by Ermak and Buckholtz [12] method of the equation in discussion, seemed to be more appropriate to our conception [3].

The differential equations (2) are required to be approximated by the difference equations [13] which compute the velocity and location of the particles, becoming equations (3) and (4). The recurrence relations of the particle's velocity and of the particle's location are

$$v(t + t_s) = v_0 (1 - e^{-\gamma t_s}) + \frac{F_0}{m\gamma} (1 - e^{-\gamma t_s}) + B_1 \quad (3)$$

$$x(t + t_s) = x_0 + \frac{1}{\gamma} \left(v + v_0 - 2 \frac{F_0}{m\gamma} \right) (1 - e^{-\gamma t_s}) + \frac{F_0}{m\gamma} t_s + B_2$$

where t_s is the time step and the terms B_1 , B_2 satisfy the equations

$$\begin{aligned} \langle B_1^2 \rangle &= 3 \frac{k_B T}{m} (1 - e^{-2\gamma t_s}), \\ \langle B_2^2 \rangle &= 6 \frac{k_B T}{m\gamma^2} \left[\gamma t_s - 2(1 - e^{-\gamma t_s})(1 + e^{-\gamma t_s})^{-1} \right] \end{aligned} \quad (4)$$

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b) Modeling Philosophies of the Transport Phenomenon

A linear polymer molecule of l length, consisting of N monomers, each of size a , ($l=Na$) is prepared to move, from a *CIS* zone to a *TRANS* zone, through a nanometer pore whose “thickness” is double of the monomer’s diameter.

The ideal pore is a part of an infinite two dimensional membrane! We consider no interactions between the polymer and the membrane, but on the other hand the polymer interaction with the pore is strong. Here we assume that as soon as the polymer enters into the pore, it is unlikely to come back, because the theoretical probability to return is very small. This assumption is justified since under experimental conditions the energy gained by a single monomer by moving through the nanopore is much larger than the thermal energy [4].

In our simulation model, the first monomer lies exactly at the pore’s entrance. Due to the existence of an external electric field, the first monomer is pulled through the nanopore from *CIS* to *TRANS* zone. In the absence of the external field, the polymer translocation is extremely slow, practically impossible. Therefore, it is of great importance to theoretically investigate the polymer transport under an electric force generated by an external electric field, $F=qE$, where q is the electric charge of monomer and E is the electric field intensity.

Results and discussions

The general Brownian motion equation for a particle under an external electric field can be easily solved by a discrete method, implementable in a versatile computing code. The programming language in which the simulation was performed is Borland C++, 3.1 version [3].

Several logical assumptions should be now made such that the transport phenomenon to be produced:

- i). the first monomer from the chain should be found at the pore’s entrance;
- ii). an external electric field should be applied to “push” the polymer through the pore from *CIS* to *TRANS* zone;
- iii). the polymer shouldn’t “tie” itself at the pore’s entrance.

Not respecting the last condition would result in blocking the translocation [3, 4, and 11].

When writing the monomers displacement equations, we have to take into account some distinct realistic conditions.

1. The monomer is found near the membrane but not in the pore’s proximity, (the *CIS* zone). In this situation the monomer is repelled by the wall through the *LJ* force’s agency.

```

if(((xp1-X[j][p])<=162.1)&&((xp1-X[j][p])>=0))
{
  if(((yp1+62.5)>Y[j][p])|(Y[j][p]>(yp2-62.5)))
  {
    r=xp1-X[j][p]-50;
    flj=Flj(r);
    Fx[j][p]=Fx[j][p]-flj;
  }
}

```

2. The monomer is inside the pore. In this situation the monomer will traverse the pore and both the superior and inferior walls will be acting (with the *LJ* repelling force) on it.

```

if(((xp1-50)<=X[j][p])&&((xp2+50)>=X[j][p]))
{
  r=yp2-Y[j][p]-50;
  flj=Flj(r);
  Fy[j][p]=Fy[j][p]-flj;

  r=Y[j][p]-yp1-50;
  flj=Flj(r);
  Fy[j][p]=Fy[j][p]+flj;
}

```

3. The monomer is found near the membrane but not in pore’s proximity, in the *TRANS* zone. In this situation the monomer is repelled by the wall through the *LJ* force’s agency.

```

if(((X[j][p]-xp2)<=162.1)&&((X[j][p]-xp2)>=0)) // near the membrane
{
  if(((yp1+62.5)>Y[j][p])|(Y[j][p]>(yp2-62.5)))
  {
    r=X[j][p]-xp2-50;
    flj=Flj(r);
    Fx[j][p]=Fx[j][p]+flj;
  }
}

```

Regarding the monomer-monomer interactions four different cases exist!

Case 1. The first monomer is fixed – the second monomer is found in first quadrant relatively to the first one.

```

if((X[j][p]<=X[j+1][p])&&(Y[j][p]>=Y[j+1][p]))
{
  sin=(Y[j][p]-Y[j+1][p])/(r+100);
  cos=(X[j+1][p]-X[j][p])/(r+100);
  Fx[j][p]=Fx[j][p]+fene*cos;
  Fy[j][p]=Fy[j][p]-fene*sin;
  Fx[j+1][p]=Fx[j+1][p]-fene*cos;
  Fy[j+1][p]=Fy[j+1][p]+fene*sin;
  Fx[j][p]=Fx[j][p]-flj*cos;
  Fy[j][p]=Fy[j][p]+flj*sin;
  Fx[j+1][p]=Fx[j+1][p]+flj*cos;
  Fy[j+1][p]=Fy[j+1][p]-flj*sin;
}

```

Case 2. The first monomer is fixed – the second monomer is found in the second quadrant relatively to the first monomer.

```

if((X[j][p]>=X[j+1][p])&&(Y[j][p]>=Y[j+1][p]))
{
  sin=(Y[j][p]-Y[j+1][p])/(r+100);
  cos=(X[j][p]-X[j+1][p])/(r+100);
  Fx[j][p]=Fx[j][p]-fene*cos;
  Fy[j][p]=Fy[j][p]-fene*sin;
  Fx[j+1][p]=Fx[j+1][p]+fene*cos;
  Fy[j+1][p]=Fy[j+1][p]+fene*sin;
  Fx[j][p]=Fx[j][p]+flj*cos;
  Fy[j][p]=Fy[j][p]+flj*sin;
  Fx[j+1][p]=Fx[j+1][p]-flj*cos;
  Fy[j+1][p]=Fy[j+1][p]-flj*sin;
}

```

Case 3. The first monomer is fixed – the second monomer is found in the third quadrant relatively to the first monomer

```

if((X[j][p]>=X[l][p])&&(Y[j][p]<=Y[l][p]))
{
    sin=(Y[l][p]-Y[j][p])/(r+100);
    cos=(X[j][p]-X[l][p])/(r+100);
    Fx[j][p]=Fx[j][p]-fene*cos;
    Fy[j][p]=Fy[j][p]+fene*sin;
    Fx[j+1][p]=Fx[j+1][p]+fene*cos;
    Fy[j+1][p]=Fy[j+1][p]-fene*sin;
    Fx[j][p]=Fx[j][p]+flj*cos;
    Fy[j][p]=Fy[j][p]-flj*sin;
    Fx[l][p]=Fx[l][p]-flj*cos;
    Fy[l][p]=Fy[l][p]+flj*sin;
}

```

Case 4. The first monomer is fixed – the second monomer is found in the fourth quadrant relatively to the first monomer

```

if((X[j][p]<=X[j+1][p])&&(Y[j][p]<=Y[j+1][p]))
{
    sin=(Y[j+1][p]-Y[j][p])/(r+100);
    cos=(X[j+1][p]-X[j][p])/(r+100);
    Fx[j][p]=Fx[j][p]+fene*cos;
    Fy[j][p]=Fy[j][p]+fene*sin;
    Fx[j+1][p]=Fx[j+1][p]-fene*cos;
    Fy[j+1][p]=Fy[j+1][p]-fene*sin;
    Fx[j][p]=Fx[j][p]-flj*cos;
    Fy[j][p]=Fy[j][p]-flj*sin;
    Fx[j+1][p]=Fx[j+1][p]+flj*cos;
    Fy[j+1][p]=Fy[j+1][p]+flj*sin;
}

```

In all the four cases of monomers interactions there appear two interaction forces, namely the attraction *FENE* force and the repelling *LJ* force [3]. The attraction *FENE* force derives from *FENE* (Finite Extension Nonlinear Elastic) potential and the repulsive *LJ* force derives from *LJ* (Lennard-Jones) potential [3].

The supplementary forces also acting on the monomers such as the viscous friction force and the external force due to the electric field action haven't been forgotten:

$$F_x[j][p]=F_x[j][p]+F_{ext};$$

$$F_x[j][p]=F_x[j][p]-\beta v_x[j][p];$$

$$F_y[j][p]=F_y[j][p]-\beta v_y[j][p].$$

The stated objective of this simulation is to evaluate the external force F and the polymer's monomers number N influences on the polymer translocation time t . The principal results of the numerical computation are presented in the following figures.

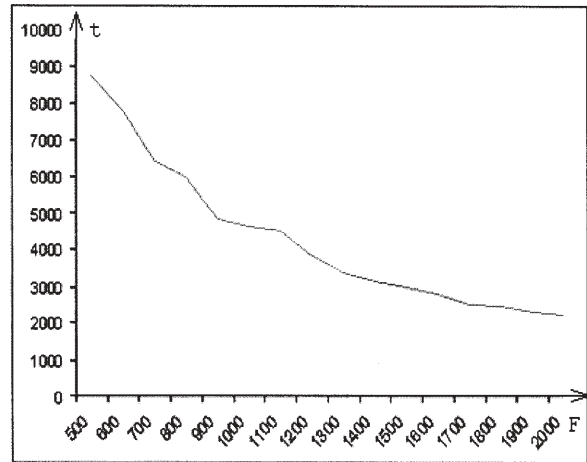


Fig. 2 The external force influence on the translocation time ($N = 16$ monomers)

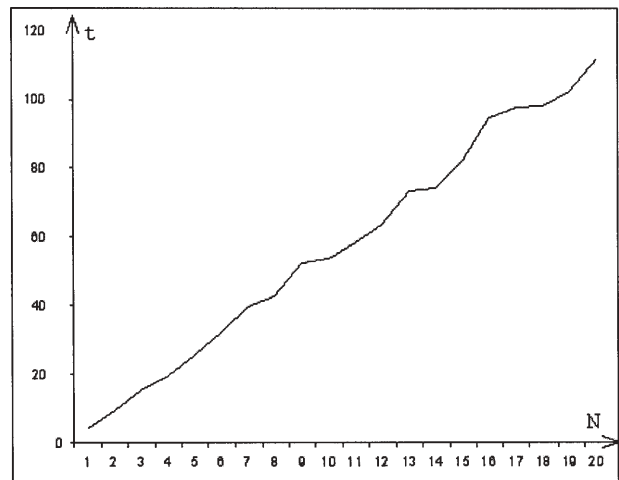


Fig. 3 The influence of the monomers number on the translocation time, at high external forces

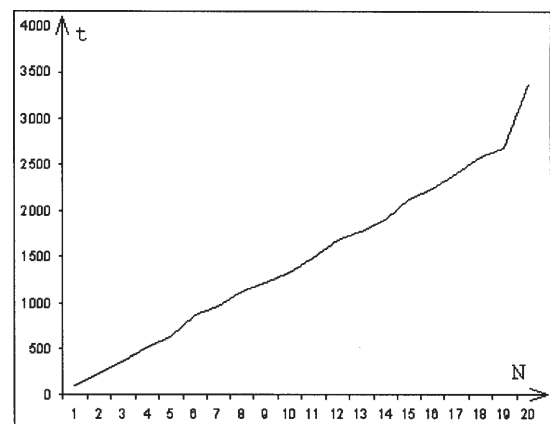


Fig. 4 The influence of the monomers number on the translocation time, at low external forces

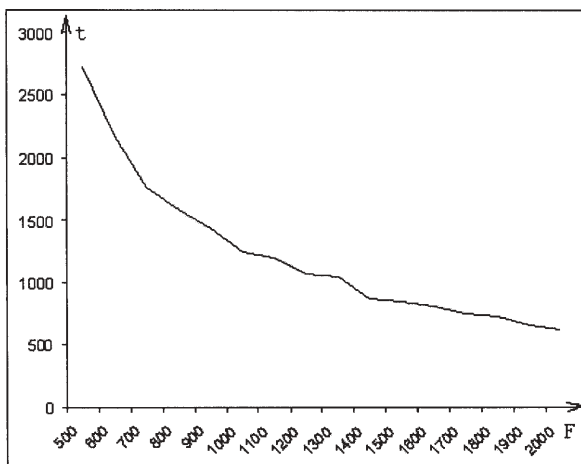


Fig. 1 The external force influence on the translocation time ($N = 5$ monomers)

Conclusions

In this paper, the polymer translocation phenomenon through a nanopore in the presence of an electric force generated by an external electric field is numerically examined. The analysis of polymers transport process through nanometer pores has confirmed the external factors deciding influence, generally known. Among these we can enumerate the pore's thickness (equal with double of the monomers diameter), the polymer's length and the measure of the external field.

The translocation time is a function which mainly depends on the polymer's monomers number, being directly proportional with the polymer's length.

The effect of the external electric field E on the translocation time is major. The force resulting from the electric field's action has an important role in the transport process. Therefore, the translocation time is inversely proportional with the electric field's intensity.

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