Viscoelastic Behaviour Analysis of the Technical Polymers by Bidimensional Pulses Generation

STEFAN PUSCA[1], MARIA-ALEXANDRA PĂUN[2]*, CRISTIAN TOMA[1]
[1] "Politehnica" University of Bucharest, Physics Department, 313 Splaiul Independentei, 060042, Bucharest, Romania
[2] "Politehnica" University of Bucharest, Faculty of Engineering in Foreign Languages, 313 Splaiul Independentei, 060042, Bucharest, Romania

This paper studies the bidimensional pulses generation in organic materials and technical polymers starting from a practical test functions basis, having as purpose their viscoelastic behaviour analysis. A heuristic algorithm for generating the test function using MATLAB procedures is presented.

Keywords: viscoelastic behaviour, test functios, polymer

The theory of hereditary elasticity or viscoelasticity is particularly widely used for polymeric materials, whose behaviour is linear or almost linear even under relatively high stresses. In the literature on rheology physics, we generally find spectral representations of the creep or relaxation functions, [1, 2]. Various molecular theories are constructed so that the resultant kernels are sums of certain number of exponential terms, [2].

The viscoelastic and viscoplastic behaviour of organic materials like bone, tendon or wood, as well as technical polymers, is largely documented. It is usually modeled using linear "Newtonian" friction, i.e., a viscous force proportional to the deformation rate. If the experimental results cannot be fitted with the resulting exponential "Debye" curves, a multitude of relaxation mechanisms or a spectrum of relaxation times is invoked.

Recent study models of the viscoelastic behaviour (well confirmed by the experimental data) use a logarithmic dependency of the logarithmic dependence of the force on the strain rate, [3]. The corresponding equation of motion is solved in the quasi-static approximation and the solutions display just the typical deviations from the Debye behaviour experimentally found, without any complications from multi-mechanism relaxation.

Further on, we will present a model of asymmetric bidimensional pulses generation and propagation simulation in organic materials and technical polymers, based on their behaviour at pulses for high frequency input signals (generated using asymmetrical test functions, [3, 4]), generalized for the bidimensional case.

We will realize the simulation in the case in which we can consider any asymmetric pulse as an element of a Hilbert Space (vectorial, normed), in which the elements of the generator system are particular classes of asymmetric functions.

The complex asymmetric bidimensional pulse formation is being made taking into consideration that on the two axes onto which the motion is being propagated, we are dealing with measures with separable variables (the basic components don't create interference effects on the two directions).

Theoretical background

The necessity of a function based approach

We observe that, if trying to realize this analysis based on the usual test functions (similar to the Dirac functions, found in the class of functions which differ from zero only on a certain limited time interval, and posses an infinite number of continuous derivatives on the real axis), we will encounter the problem that they can't be generated (as we know) by a differential equation.

The analysis should be limited to practical test functions usage (12), defined as functions that differ from zero on a certain interval and posses a finite number of continuous derivatives on the whole real axis.

The advantage offered by the present model usage is given by the fact that, due to the phenomena taking place in a spatially limited area, this doesn't require the use of propagation equations, the results obtained being independent of these.

Basic properties of practical test-functions used for generating asymmetrical pulses in organic materials and technical polymers

Frequently the analysis of signals on limited time intervals requires the use of adequate mathematical models able to generate alternating function. For obtaining sinusoidal type fonctions able to generate signals with a certain angular frequency, the undumped differential equations of second order are well know.

But for obtaining pulses limited on certain time intervals some specific models must be used. An alternative is represented by the use of test-functions, but ideal test functions can not be generated by a differential equation of evolution of the type presented in paper [3].

On the the other side, a propagation phenomenon for an ideal test function can not be taken into consideration as in [4], because we are looking for causal pulses, generated in a rigorous manner by an equation of evolution. This implies the use of practical test-functions (functions which possess a limited number of derivatives equal to zero at the limits of the working interval and which can be solutions of differential equations).

So we must study invariance properties of such equations, so for the output to be represented by a function asymmetrical as related to the middle of the working period.

If we consider as working interval (-1, 1)the time interval, then the middle of the interval would be the origin, and the condition for the output f(t) to be asymmetrical corresponds to the condition

$$f(-t) = -f(t)$$

* maria_paun2003@yahoo.com

For obtaining such a function on the time interval (-1, 1), we must begin by studying equations able to generate symmetrical function *g* on this time interval so as to find a method for translating some of their properties to asymmetrical functions; finally, we must use Runge-Kutta equations,[5], for studying the properties of the mathematical models obtained.

We are looking for controlled oscillations on a limited time interval (unlike unstable oscillations for second order difference systems).

This would lead to a sine or cosine function, with possibilities of joining together such working intervals for obtaining a controlled oscillation extended in time.

Yet we are looking for general differential equations able to generate asymmetrical pulses of different shapes (not only sine or cosine functions). We can extend our analysis at wavelets corresponding to PDE [6] or to equations able to generate wavelets represented by solitary waves [7].

Both previously mentioned aspects can be joined together if we are looking for functions similar to test-functions having a shape similar to wavelets.

As can be noticed studying [5], practical test-functions of second order possess a derivative with null initial and final values. Analyzing its mathematical expression on the whole working interval, we can notice that, this derivative is an asymmetrical function as related to the middle of the working interval (considered as origin), while the symmetry of g function implies that its slope is asymmetrical as related to the origin (the same modulus and opposite sign).

So we must analyze the differential equations able to generate symmetrical functions *g* and to study the shape of their derivatives for different input, functions.

As it is known, a *test-function* on [a, b] is a C^∞ on R which is nonzero on (a, b) and zero elsewhere. For example the bump-like function

$$\varphi_a(\tau)=\begin{cases}\exp\left(\dfrac{1}{\tau^2-1}\right), & if \quad \tau\in(-1,1)\\ 0, \quad if \quad \tau\notin(-1,1)\end{cases} \tag{1}$$

is a test-function on $[-1,1]$.

On the other side, test-function as the bump-like function

$$\varphi_b(\tau)=\begin{cases}\exp\left(\dfrac{0,1}{\tau^2-1}\right), & if \quad \tau\in(-1,1)\\ 0, \quad if \quad \tau\notin(-1,1)\end{cases} \tag{2}$$

is almost equal to a constant value for 2/3 of the working period (similar to a step-function)

Such functions can not be generated by differential equations of evolutions; we can use differential equations able to generate a practical test-function on (-1,1)(a C^n nonzero function on (-1, 1) which satisfies the boundary conditions $f^{(k)}(a)=f^{(k)}(b)=0$ for k=0,1,...,n and can be a solution of an initial value problem on this interval).

The first and second derivatives of φ_a are:

$$\varphi_a^{(1)}(\tau)=\frac{-2\tau}{\left(\tau^2-1\right)^2}\exp\left(\frac{1}{\tau^2-1}\right)$$

$$\varphi_b^{(1)}(\tau)=\frac{6\tau^4-2}{\left(\tau^2-1\right)^2}\exp\left(\frac{1}{\tau^2-1}\right). \tag{3}$$

And by simply dividing the function φ_a a(τ) to $\varphi^{1}_{(a)}$ (τ) we obtain:

$$\varphi_a^{(1)}=\frac{-2\tau}{\left(\tau^2-1\right)^2}\varphi_a. \tag{4}$$

Considering the corresponding differential equation:

$$f^{(1)}=\frac{-2\tau}{\left(\tau^2-1\right)^2}f \tag{5}$$

with initial values considered at τ_o = 0.99 as:

$$f_0=\exp\left(\frac{1}{0,99^2-1}\right). \tag{6}$$

It results a function *f* symmetrical as related to the middle of the working interval.

Its derivative, a saw-teeth asymmetrical pulse, is represented in figure 1.

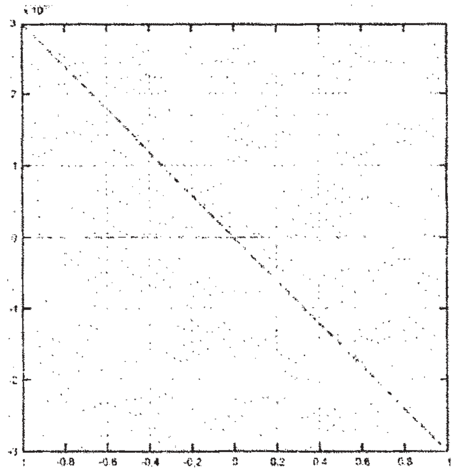

Fig.1. The function derivative (a saw-teeth asymmetrical pulse)

The same way, the correspondence between φ_a and $\varphi^{(2)}_{(a)}$ results under the form:

$$\varphi_a^{(2)}=\frac{6\tau^4-2}{\left(\tau^2-1\right)^4}\varphi_a \tag{7}$$

By considering the corresponding differential equation:

$$f^{(2)}=\frac{6\tau^4-2}{\left(\tau^2-1\right)^4}f \tag{8}$$

with initial values considered at τ_o =-0,99 as:

$$f_0=\exp\left(\frac{1}{0,99^2-1}\right) \tag{9}$$

$$f_0^{(1)}=\left[2\frac{0,99}{\left(0,99^2-1\right)^2}\right]\exp\left(\frac{1}{0,99^2-1}\right) \tag{10}$$

it results also a function *f* symmetrical as related to the middle of the working interval.

Its derivative, an asymmetrical pulse, is represented in figure 2.

In a similar way, for the function $\varphi_\beta(\tau)$ we obtain the correspondence:

$$\varphi_b^{(2)}=\frac{0,6\tau^4-0,36\tau^2-0,2}{\left(\tau^2-1\right)^4}\varphi_b \tag{11}$$

By considering the corresponding differential equation:

$$f^{(2)}=\frac{0,6\tau^4-0,36\tau^2-0,2}{\left(\tau^2-1\right)^4}f \tag{12}$$

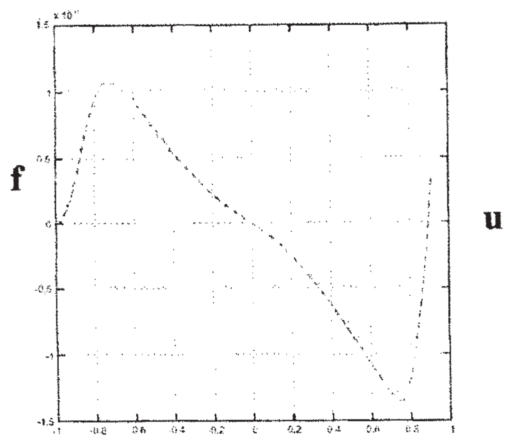

Fig.2. The function derivative (an asymmetrical pulse)

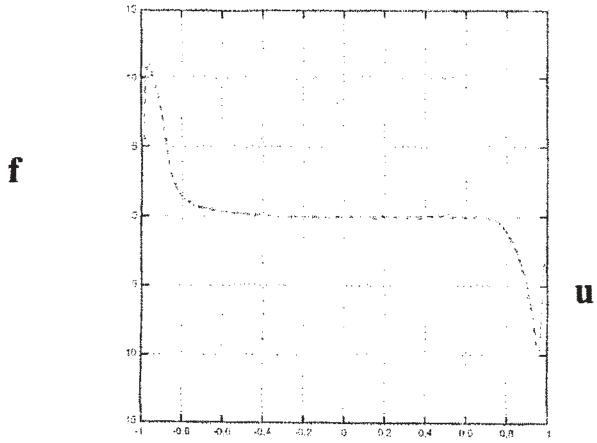

Fig. 3. The function derivative (a sharp asymmetrical puls)

with initial values considered at τ_0=-0.99 as:

$$f_0 = \exp\left(\frac{0,1}{0,99^2-1}\right) \tag{13}$$

$$f_0^{(1)} = \left[0,2\frac{0,99}{(0,99^2-1)^2}\right]\exp\left(\frac{0,1}{0,99^2-1}\right) \tag{14}$$

we obtain a function *f* symmetrical as related to the middle of the working interval.

Its derivative, a sharp asymmetrical pulse, is represented in figure 3.

The shape of these outputs offers also the possibility of joining together such time intervals and the corresponding asymmetrical pulses so as to obtain a controlled oscillation.

While at the beginning and the end of each working interval the state-variables of the differential system are approximately equal to zero it would be quite easy to adjust the final values of these variables for a working-interval to the initial values of these variables for the next working interval: thus the cycle can continue in a controlled manner.

Results and discussions

We must point the fact that an asymmetrical pulse represents in fact a test function for the derivative of an input signal. By multiplying an input signal with an asymmetrical pulse and by integrating the resulting function on the working interval, we obtain a result proportional to the slope of the input signal (as it can be easily checked). The method is presented in [8], where the input function is processed by a nonlinear second order system.

Thus, for asymmetrical pulse generation of any shape we can see that the pulses presented in the previous three examples form a system of generators in the space of the pulses on one of the axes, so any pulse can be written under the form:

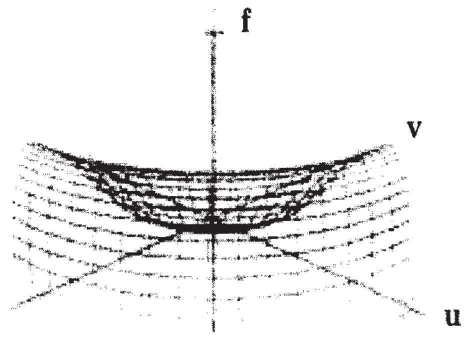

Fig. 4. Bidimensional wave wrapping for the figure 1 case

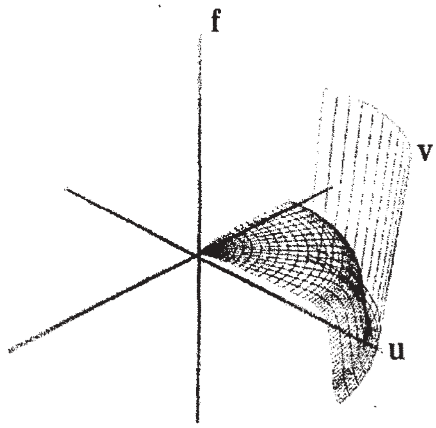

Fig.5. Bidimensional wave wrapping for the figure 2 case

$$f_k^{(j)}(\tau) = \sum_{i=1}^{3} \varphi_i^{(j)}(\tau)\cdot C_{i,\alpha}^{(j)} \tag{15}$$

(where $C_{i,\alpha}$ are the expansion coefficients, to be determined in the concrete conditions of the pulse to be generated).

Generalization for the bidimensional case

When we are interested in generating bidimensional pulses in organic materials [8], or studied technical polymers profiles we can consider, as we have remembered [9], the case in which the behaviours on the two directions don't appreciably interfeer, so we obtain for the three presented situations the following bidimensional wave wrappings (in which we have on the axes -*ox* and -*oy* the measures ***u*** and ***v*** respectively and on the -*oz* axis we have ***f***)

We can generate any bidimensional pulse with the help of the system of generators (overdimensioned – in the case of our simulations realized in MATLAB) in the following way:

$$F_k(\tau) = \sum_{j=1}^{2} e_j f_j(\tau) \tag{16}$$

where e_j are the versors of the two directions onto which we can generate the pulses, and

$$f_j(\tau) = \sum_{i=1}^{3} \varphi_i^{(j)}(\tau) C_{i,\alpha}^{(j)}. \tag{17}$$

At the end, we have to study the output signal generated by a second order differential equation, able to generate an asymmetrical output signal, corresponding to an input signal represented by a very short pulse, so that its stability to this kind of perturbations can be checked.

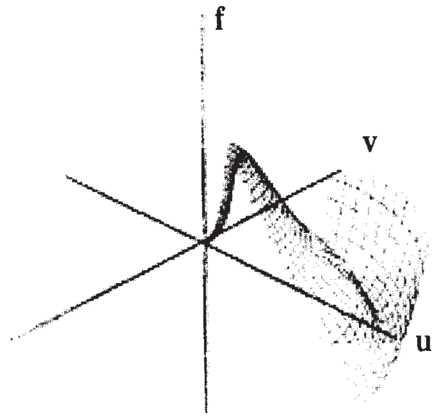

Fig. 6. Bidimensional wave wrapping for the figure 3 case

For this, we consider the differential equation:

$$f^{(2)} = \frac{0,6\tau^4 - 0,36\tau^2 - 0,2}{\left(\tau^2 - 1\right)^4} f + 0,1\exp\left[-\frac{\left(x+0,9\right)^2}{0,01^2}\right] \quad (18)$$

with zero initial conditions (the external pulse being represented by a short Gaussian pulse received at the moment $t_p = -0,9$). The derivative of this output signal is represented in figure 7.

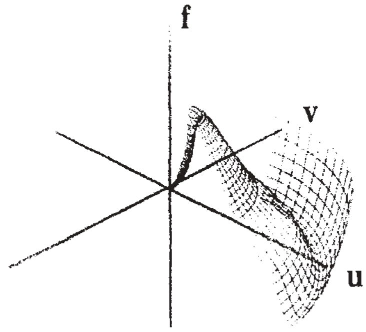

Fig. 7. The derivative of the output signal

The final bidimensional representation of the final output signal wrapping for the organic material studied is, according to the MATLAB simulation, the one in figure 8.

Conclusions

This work presents a model of asymmetric bidimensional pulses generation and propagation simulation in organic materials and technical polymers, based on their behaviour at pulses for high frequency input signals, generalized for the bidimensional case.

Taking into discussion the action of the pulse on a direction, we obtain as a first consequence, a possible application, the more rapid estimation of the acceleration

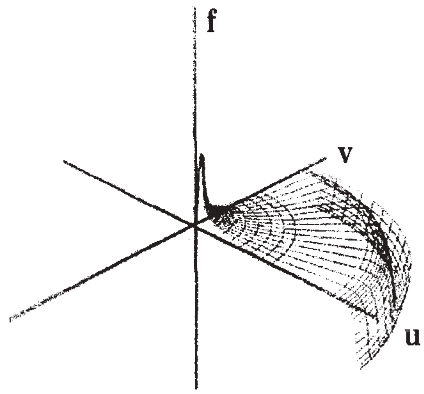

Fig. 8. Final output signal wrapping for the organic material

by modification of the input signal, corresponding to the velocity.

Another application can be represented by the phase detection, by multiplication of the alternate input signal with an assymetrical function and then integrating the resulting function, we obtain a result proportional with the amplitude of a sinusoidal input function.

It can be observed, in figure 7, that a major influence appears at the end of the interval after a time of approximatively 1.8 units. So, the final output signal can be considered as a noncausal pulse from the perspective of an external observer which studies the input and output system signals on the interval $(0,1)$ for example. As opposed to the aspects regarding the noncausal waves as possible solutions of the wave equation, the noncausal pulse generated by the previous equation appears with approximative zero conditions, existing for the state variables of an unique system (a single point).

The term "approximative zero conditions" implies the use of a phenomena multilevel analysis.

References

1. PĂUN, V.-P., Mat. Plast. , **40**, nr. 1, 2003, p. 25
2. PĂUN, V.-P., Mat. Plast., **40**, nr. 2, 2003, p. 81
3. JAGER, I. L., J Biomech. , **38** (7), 2005, p.1451-8
4. VLAD M.O., SCHRONFISCH B., MACKLEY M.C., Physics A 229, 1996, p. 312-342
5. PUSCA, S., Lect Notes Comput SC 3980, 2006, p. 763
6. CATTANI, C., Computers and Mathematics with Applications, **50**, 2005, p. 1191
7. RUSCHCHITSKY, J.J., CATTANI, C.,TERLETSKAYA, E.V., International Applied Mechanics, **40**, 3, 2004, p. 3118
8. LAZĂR, B., STERIAN, A., PUŞCĂ, S., PĂUN, V.-P., TOMA, C., MORARESCU, C., Lect Notes Comput SC 3980 , 2006, p. 779
9. IORDACHE, D., PUŞCĂ, S., TOMA, G., PĂUN, V.-P., STERIAN, A., MORARESCU, C., Lect Notes Comput SC 3980, 2006, p. 804

Manuscript received: 11.01.2007

After 5 h some peaks appear different from natural frequencies. Some of them disappear in final frequency spectrum, but new appears again. This phenomenon is determined by sediments resulted in grease degradation process. They alter frequency spectrum just like race waves but, there is a difference, because they appear and disappear in running process. In order to be able to distinguish the frequencies spectra disturbances in tables 4 and 5, the natural frequencies due to inner ring rotation and the bearings race waves natural frequencies respectively are presented.

Conclusions

Degradation grease phenomenon is a very complex one. It has main influences to reliability and vibration level of ball bearings lubricated "for life".

Time domain signal processing, reflects very clear degradation process using diagnosis value K (t).

Frequency domain analysis shows degradation process through sediments which adhere and detach from race ways. These sediments generate new peaks in frequency spectrum.

Grease degradation process study using vibration tests represents one of the future possibilities to detect greases reserve life.

References

1.GRIGORAS, ST., ŞTIRBU, C., ASME-GREEK SECTION, Greece, 2001, p.50

2.GAFITANU M., IANUS G., TARABOANTA Fl., Bul. St. Univ. de Nord Baia Mare, 2003 p.173

3.MORIUKI, T., KAGEYAMA, H. LUBRICATING, JSLE International Tribology Conference, 1985, Japan, p. 1027

4.MAS, R., MAGNIN, A., Journal of Tribology nr.118 , 1996, p. 681

5. PRASHAD, H., Wear, 198, 1996, p. 46

6. PRASHAD, H., Wear, 176, 1994, p. 151

7. KOMATSUZAKI, S., UEMATSU, T., NAKANO, F., Lubrication Engineering, **43**, 1987, p. 25

Manuscript received: 16.02.2007