Buckling Analysis of a Thin-walled Cylindrical Shell Strengthened by Fiber - reinforced Polymers

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In this paper the buckling problem is solved for the laminated composite structures from a thin-walled metallic cylindrical shell which is strengthened by fiber-reinforced polymers. The research concerning the influence of composite thickness and fiber orientation on axial critical buckling load is executed. For linear buckling problem, the purposed method is based on modified relations for an orthotropic cylinder. The method was verified by comparison with buckling theories of laminated composite shells and with numerical results based on finite element method.

Keywords: lamina/ply, buckling, analytical modelling, numerical analysis

Buckling analysis is one of the main stages in the design of thin-walled structural elements for which, due to their slenderness, buckling failure is one of the most common modes of failure. Constructions from laminated composite shells are attracting the particular interest because they have wide applications in the aircraft building, automobile industry, etc.

The buckling problem of composite structural elements is considered in literatures [4 – 12]. It should be noted that the mathematical relationships of buckling analysis get more complex when researcher's attention is shifting from consideration of homogeneous orthotropic structures to multi-layered composite from heterogeneous materials with various angles of fiber orientation.

Another important thing in the buckling problem of composite structures is the question about the most efficient reinforcement of composite material for the definite loads because one feature of FRP composite structures is the possibility to modify in wide range their deformation and strength properties by changing the fibers orientation.

In this paper the authors propose one method of buckling analysis of laminated cylindrical shell which is based on the stability equation of an orthotropic cylinder with modified relations for bending and extensional stiffnesses. In this equation, the stiffnesses were calculated for the reinforced cylindrical shell which is modeled as a bi-layer lamina, where the isotropic ply is one layer and the reinforced ply another layer. The results obtained by this method are compared with the theory of stability calculation for laminated composite structures and with numerical results of finite element method. Research about the influence of fiber orientation in FRP on buckling of considered construction is also presented.

Statement of the problem

The cylindrical shell strengthened by FRP can be modeled like two coaxial cylindrical shells which are glued without interference fit and with the ideal contact conditions (fig.1a), where the notations used are: L – length of the shells, R, t_s – radius and thickness of the interior shell, t_c – thickness of the external shell. The interior shell is made from isotropic material (steel) with elastic modulus E and Poisson's ratio μ, and the external shell – from several FRP bilayers. Each bilayer is formed from two monolayers with mechanical characteristics E_1, E_{2f}, μ_{12f}, G_{12f} which are orientated by $\pm\theta$ (fig.1b). This provides that the principal directions of orthotropic material for the external shell coincide with the cylindrical coordinate system. The resulting properties for the external shell are E_1, E_2, μ_{12}, G_{12} and they are function of monolayer properties and θ. The structure is simply supported and axially compressed.

The analysis performed has the following goals:

- to determine the influence of composite thickness on the axial critical buckling load;
- to determine the influence of fiber orientation in polymer

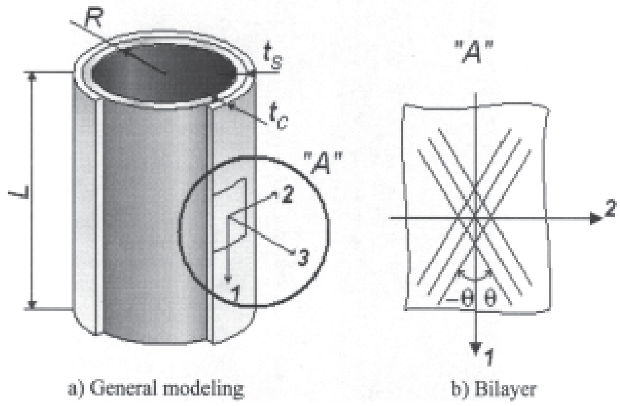

Fig. 1. Cylindrical shell strengthened by composite material

Theoretical background

Analytical solutions available [6] and [7], called in this paper M.L.1 and respectively M.L.2, used to calculate critical buckling loads for axially compressed, thin-walled laminated orthotropic composite cylinders, are based on the use of macromechanics of laminate composite. Macromechanics is the study of a laminate response to loading, based on the properties of each lamina and the stacking sequence. Classical lamination theory permits to determine the stiffness of a laminate if the properties and orientation of each lamina (layer) in the laminate is known. The stiffness matrices are needed in order to analyze a laminate under a given loading condition.

In this article, we propose an analytical formulation for the buckling problem, by treating laminated shell as a

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single layer, but with recalculated stiffnesses and material properties.

The present method is based on the use of theory presented in [5], where the stability equation for an orthotropic shell subject to axial compression is:

$$P=\left(\frac{m\cdot\pi}{L}\right)^2\left[\frac{D_x}{1-\mu_x\cdot\mu_y}+\left(\frac{2\mu_y\cdot D_x}{1-\mu_x\cdot\mu_y}+2\cdot D_{xy}\right)\left(\frac{n\cdot L}{m\cdot\pi\cdot R}\right)^2+\frac{D_y}{1-\mu_x\cdot\mu_y}\left(\frac{n\cdot L}{m\cdot\pi\cdot R}\right)^4\right] + \frac{E_x\cdot E_y}{\left(\frac{m\cdot\pi}{L}\right)^2\cdot R^2\cdot\left(E_x-\left(2\mu_y\cdot E_x-\frac{E_x\cdot E_y}{G_{xy}}\right)\cdot\left(\frac{n\cdot L}{m\cdot\pi\cdot R}\right)^2+E_y\cdot\left(\frac{n\cdot L}{m\cdot\pi\cdot R}\right)^4\right)} \quad (1)$$

To compute the buckling load from equation (1), where the axial load appears as a function of number of axial half-waves m and number of circumferential full waves n, this load must be minimized for integral values of both m and n.

In equation (1), the notations used are: L - length of cylinder, R mean radius of cylinder, t thickness of the wall, μ_x, μ_y Poisson's ratios in longitudinal and circumferential directions, respectively, E_x, E_y extensional stiffnesses of orthotropic shell in longitudinal and circumferential directions, respectively, defined by equations (2) and (3), D_x, D_y bending stiffnesses of orthotropic shell in longitudinal and circumferential directions, respectively, defined by equations (4) and (5), G_{xy} in-plane shear stiffness of orthotropic shell defined by equation (6); D_{xy} twisting stiffness of orthotropic shell defined by equation (7):

$$E_x = E_1\cdot t \quad (2)$$

$$E_y = E_2\cdot t \quad (3)$$

$$D_x=\frac{E_1\cdot t^3}{12} \quad (4)$$

$$D_y=\frac{E_2\cdot t^3}{12} \quad (5)$$

$$G_{xy}=G_{12}\cdot t \quad (6)$$

$$D_{xy}=\frac{G_{12}\cdot t^3}{6} \quad (7)$$

In the above equations, E_1 represents the Young's modulus of composite in the meridional direction; E_2 represents the Young's modulus of composite in the circumferential direction and G_{12} is the shear modulus of composite material.

In case of metallic shell strengthened with composite material, was used the expression (1), but some approximations were made.

Firstly, the parameters expressed by (2), (3) and (6) were recalculated for a shell that has two layers. The approximations made were as follows:

$$E_{x,m}=\frac{E\cdot t_s+E_1\cdot t_c}{t_s+t_c}\cdot(t_s+t_c) \quad (8)$$

$$E_{y,m}=\frac{E\cdot t_s+E_2\cdot t_c}{t_s+t_c}\cdot(t_s+t_c) \quad (9)$$

$$G_{xy,m}=\frac{G\cdot t_s+G_{12}\cdot t_c}{t_s+t_c}\cdot(t_s+t_c) \quad (10)$$

In order to estimate the bending and twisting stiffness, was determined the position of the neutral surface (the distance z^* from the lower edge of the shell) for the shell with two layers made from different materials. Because of the bending stresses, the middle plane does not deform.

The position of the middle plane (neutral surface) is obtained from condition that the bending moment by neutral axis to be zero.

$$M_{z^*}=\int_{-z^*}^{t_s+t_c-z^*}\sigma_x\cdot z dz=\int_{-z^*}^{t_s-z^*}\sigma_x\cdot z dz+\int_{t_s-z^*}^{t_s+t_c-z^*}\sigma_x\cdot z dz=0 \quad (11)$$

The curvatures of the neutral surface in sections parallel to the zx and yz planes are:

$$\chi_x=\frac{1}{\rho_x}=-\frac{\partial^2 w}{\partial x^2} \quad (12)$$

$$\chi_y=\frac{1}{\rho_y}=-\frac{\partial^2 w}{\partial y^2} \quad (13)$$

The unit elongations in the x and y directions of a thin lamina at a distance z from the neutral surface (in the case of a thin shell and taking into account that normal strain of the neutral surface is equal to zero) can be found as [10]:

$$\varepsilon_x=\frac{z}{\rho_x} \quad (14)$$

$$\varepsilon_y=\frac{z}{\rho_y} \quad (15)$$

From Hooke's law, the corresponding x normal stress is:

$$\sigma_x=\frac{E(z)}{1-\mu^2}(\varepsilon_x+\mu\varepsilon_y)=\frac{E(z)\cdot z}{1-\mu^2}\left(\frac{1}{\rho_x}+\mu\frac{1}{\rho_y}\right) \quad (16)$$

where Young modulus is different for each layer and it should be taken:

$$E(z)=\begin{cases}E, 0\le z\le t_s\\ E_1, t_s\le z\le t_s+t_c\end{cases} \quad (17)$$

By introducing (16) and (17) in (11), the position of the neutral surface results:

$$z^*=\frac{E\cdot t_s{}^2+E_1\cdot t_c{}^2+2\cdot E_1\cdot t_s\cdot t_c}{2(E\cdot t_s+E_1\cdot t_c)} \quad (18)$$

Considering the equation (16), the bending moment per unit length of the edges parallel to the x-axis, M_x can be expressed as:

$$M_x=\left(\frac{1}{\rho_x}+\mu\frac{1}{\rho_y}\right)\cdot\left[\int_0^{t_s}\frac{E(z)\cdot(z-z^*)^2}{1-\mu^2}dz+\int_{t_s}^{t_s+t_c}\frac{E(z)\cdot(z-z^*)^2}{1-\mu^2}dz\right] \quad (19)$$

On the other hand, M_x can be written as:

$$M_x=\frac{D_x}{1-\mu^2}\left(\frac{1}{\rho_x}+\mu\frac{1}{\rho_y}\right) \quad (20)$$

Equating (19) and (20), the bending stiffness in longitudinal direction for the shell with two layers is obtained:

$$D_x=\frac{1}{3}\left\{E\left[\left(t_s-z^*\right)^3+\left(z^*\right)^3\right]+E_1\left[\left(t_s+t_c-z^*\right)^3-\left(t_s-z^*\right)^3\right]\right\} \quad (21)$$

In the same manner is obtained D_y:

$$D_y=\frac{1}{3}\left\{E\left[\left(t_s-z^*\right)^3+\left(z^*\right)^3\right]+E_2\left[\left(t_s+t_c-z^*\right)^3-\left(t_s-z^*\right)^3\right]\right\} \quad (22)$$

In order to obtain twisting stiffness D_{xy}, the torsional moment M_{xy} is expressed as:

$$M_{xy} = \int_0^{t_s+t_c} \tau_{xy}\cdot(z-z^*)dz = \int_0^{t_s+t_c} 2\cdot G(z)\cdot(z-z^*)^2\cdot\chi_{xy}dz \quad (23)$$

where τ_{xy} represents the shearing stress, χ_{xy} is the twist of element during bending of the shell and shear modulus is different for each layer and it should be taken:

$$G(z) = \begin{cases} G, 0 \le z \le t_s \\ G_{12}, t_s \le z \le t_s + t_c \end{cases} \quad (24)$$

On the other hand, M_{xy} is:

$$M_{xy} = \frac{D_{xy}}{1-\mu^2}\cdot(1-\mu)\cdot\chi_{xy} \quad (25)$$

Equating (23) and (25), the twisting stiffness for the shell with two layers is obtained:

$$D_{xy} = \frac{2}{3}\cdot\{G[(t_s-z^*)^3+(z^*)^3]+ +G_{12}[(t_s+t_c-z^*)^3-(t_s-z^*)^3]\} \quad (26)$$

Results about influence of composite thickness on the axial critical buckling load when the fibers orientation coincide with the load direction

The analyzed shell had a height L = 1500 mm, radius R = 1734 mm and constant thickness t_s = 3 mm. The composite sheet was considered to be glass epoxy, whose properties were E_1 = 60 GPa, E_2 = 7.3 GPa, G_{12} = 7.38 GPa and μ_{12} = 0.3.

The critical buckling load was calculated by using the classical laminated plate theory, numerical analysis and present theory, in order to see if the proposed results obtained are in agreement with the results obtained by classical laminated plate theory.

The composite thickness was varied from 0 to 15 mm, so that to estimate the influence of composite sheet on the strength of cylindrical shell, from the instability point of view.

ANSYS 12 was used to carry out the finite element analysis in the work. The element chosen was SHELL281 and it were considered two layers. The first layer was modeled as an isotropic material having steel properties and the second layer, representing the composite sheet attached to the outside surface of the shell, was modeled as an orthotropic material. The shell nodes were offset to the location z^*, so that the load to be applied on the neutral surface.

Figures 2 and 3 are presenting a comparison between the results obtained from the application of the analytical and numerical models.

The structure was supported at the bottom, restricting displacements in the three directions, and at the top of the shell only the axial displacement was free. The linear buckling analysis was used to estimate the critical buckling load.

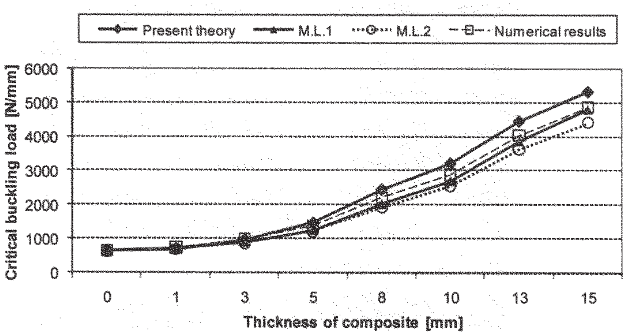

Fig. 2. Results on the critical buckling load, obtained by numerical analysis and different analytical models

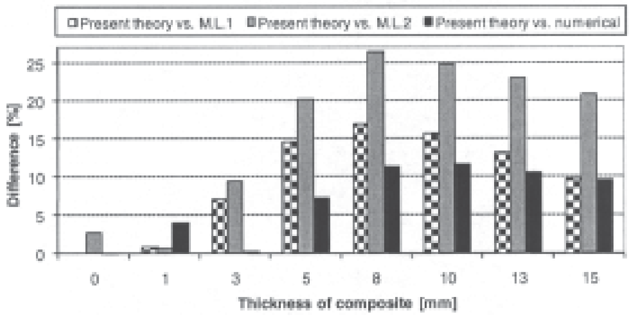

Fig. 3. Comparison between the proposed method and analytical and numerical models used for buckling analysis of steel shells strengthened with composite materials

The results of the computations presented in figures 2 and 3, show that the values of critical load obtained by present theory are very close to the ones obtained by classical laminated plate theory and numerical analysis, especially when the composite sheet thickness is small. This means that the proposed method is limited for very small R/t_c ratios.

Investigation about the influence of fibers orientation on the axial critical buckling load

Fibers orientation is essential for the physical properties of composite materials. These properties influence the shell stiffness and therefore, the critical buckling load. In order to see the influence of fiber orientation on the axial critical buckling load, it was considered that the composite sheet had two layers: one having +θ orientation, and the second having –θ orientation. The angle orientation θ was varied from 0 to 90 degrees and the physical properties of composite materials were recalculated in function of this angle [7].

The analytical and numerical calculations were made by considering t_s = 3 mm and t_c having value 3 mm (fig. 4), respectively 10 mm (fig. 5) and different fibers orientation.

The results obtained (fig. 6) show a good agreement between the proposed method, classical laminated plate

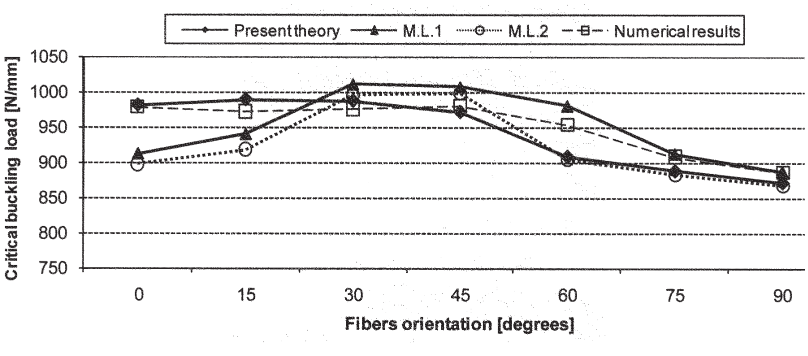

Fig. 4. Critical buckling load for different fibers orientation (t_c = 3mm)

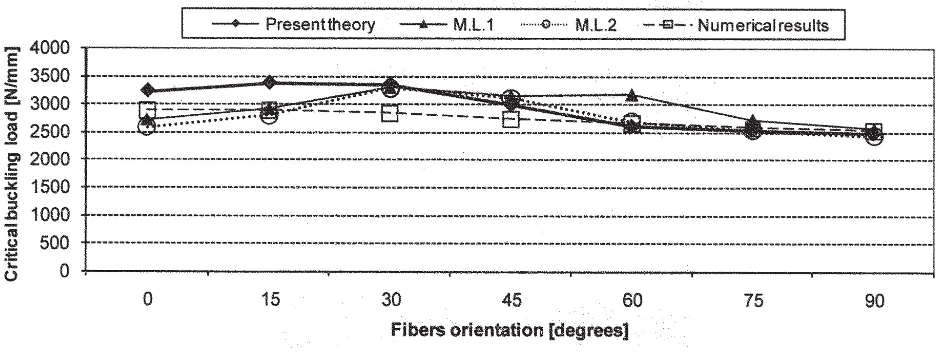

Fig. 5. Critical buckling load for different fibers orientation (t_c = 10 mm)

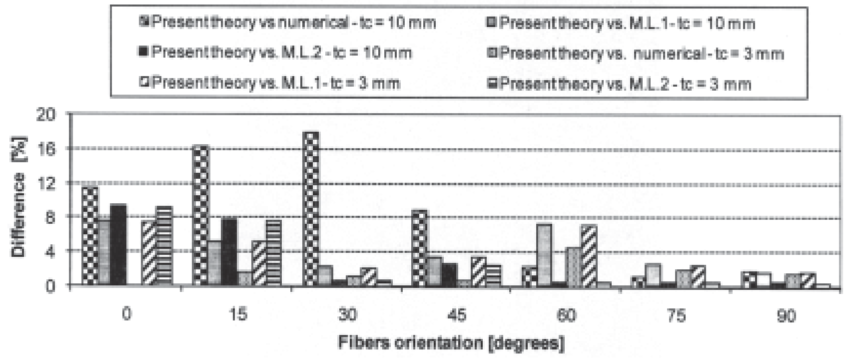

Fig. 6. Comparison between analytical and numerical results

theory and numerical results, especially for angle orientation values greater than 45 degrees.

Conclusions

The buckling analysis was done for the simply supported axially compressed metallic cylindrical shell strengthened by FRP. The proposed method, which is based on treating the multi-layered cylindrical shell as a single layer with modificated stiffnesses, gives results closer to the classical laminated plate theory, for small values of composite thickness. This method is limited for small R/t_c ratios.

Changing of fibers orientation in composite material considerably influences the axial critical buckling load. The results show that the value of critical load can be changed on 10% for the composite thickness t_c = 3 mm and on about 29 % for t_c = 10 mm. The most efficient fibers orientation for the compressive load is θ equal about 30°.

All the presented results demonstrate that the application of a composite sheet on the external surface of a thin-walled cylindrical under axial compression is a very efficient method of stiffening, leading to a significant increase in critical buckling load.

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