# Thermal Expansion Coefficient Determination of Polymeric Materials using Digital Image Correlation

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*Applications of digital image correlation (DIC) method to material characterisation has been proven to be a powerful tool for deformations and strain analysis and found widespread use and acceptance in the field of experimental mechanics. This paper describes the potential, accuracy and limitations of a commercial DIC system to full-field, real-time characterisation of the coefficient of thermal expansion (CTE) of polymeric materials. The topics such as strain calculation procedure and influence of a small rigid body rotation were theoretically described and experimentally verified. A series of measurements was carried out to determine the CTE polypropylene (PP) and polyvinylchloride (PVC) commercial plastics. To check the feasibility of the method an aluminium sample was initially analysed. The measuring set-up developed includes a simple heating device, thermal sensors and a thermo-camera for real-time temperature measurement and monitoring of the sample and a 3D-DIC measuring system. The results revealed that the DIC can be a reliable tool for thermal deformations measurement especially suitable for polymeric materials with a higher CTE.*

*Keywords: digital image correlation, thermal strains, polymeric materials*

Knowledge of real material parameters is nowadays a condition for numerical simulations, widely used in industry to design quality components in very short time. The value of the thermal expansion coefficient (CTE) plays a key role in components design, in structure response or in the final decision-related materials selection. It is also desirable to be able to visualize mechanical behaviour of materials or components under thermal loads. In the case of plastics these needs are particularly great because of the rapid change within the industry. The plastics in use today are very often not precisely the same as in the past, even if the polymer is basically the same, and there are continuing refinements in processing. Also, plastics are being used in more and more new applications, and frequently more critical applications, than before.

Conventional experimental procedures relying on dilatometers to measure CTE require a set of standard bars. Strain gauge technique can also accurately measure the thermal strain [1] but the measurements are limited to temperatures up to about 120°C with standard strain gauges and free expansion of samples are affected by the wires, disturbing the measurement. In some applications [2] these are not a reliable or easy to apply solutions. Full-field optical techniques like Electronic Speckle Pattern Interferometry (ESPI) proved to be well suited to analyse the thermal expansion behaviour and to determine the coefficients of thermal expansion (CTE) for thermally isotropic as well as anisotropic solid materials [2]. The overall accuracy of the CTE measurement by the ESPI method was estimated at $\approx 0.1 \times 10^{-6}$ [1/K]. The drawbacks of ESPI methods consist in its high sensitivity to environmental conditions such as vibrations, thermal convection currents flowing around the specimen and complexity of the measurement set-up.

The digital image correlation technique (DIC) has proven to be a useful tool for contour and deformations analysis of polymeric materials [3, 4] and found in the last years widespread use and acceptance. In the field of thermal proprieties of materials DIC was used to determine and the CTE of thin films [5] or tubular steel specimens [6]. A study regarding thermal deformations at high temperature measured by DIC and determination of chromium nickel stainless steel CTE in the temperature range 20-550°C is presented in [7]. The method of DIC is demonstrated to be capable of providing accurate measurements up to 1100°C, and the potential to monitor strains to 1400°C was identified [8]. The capability of the method was demonstrated by measuring the Young's modulus and coefficient of thermal expansion of a nickel-base superalloy at temperatures from ambient to 1000°C [8]. Compared to other methods the DIC enjoys besides of advantages being non-contact and full-field measurement method, to have a simple set-up, low sensibility to environmental conditions, easy post-processing of the measured data, no limits on the temperatures and strains than can be reached.

In this paper it is investigated the possibility and accuracy of CTE measurement using a 3D digital image correlation system for two commercial plastics: polypropylene (PP) and polyvinylchloride (PVC). The work discusses the CTE calculation of isotropic and anisotropic materials, influence of a small rigid body rotation and analyses the accuracy of strain calculation procedure. In the first part the DIC procedure is explained and the experimental set-up is outlined. The noise contained in the calculated displacements, especially when small displacements are measured, will be amplified if the strains are calculated by numerical differentiation of the displacement field. The possibilities to get more precise and reliable strain results are discussed in the paper and comparative values in case of an aluminum sample are presented. An assessment of the error on the obtained CTE is made at the end of the paper.

## Digital image correlation

In the 3D digital image correlation technique, random gray value dot patterns on specimen surfaces are observed by two cameras from different directions in a stereoscopic

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setup (fig. 1A), the position of each object point being focused on a specific pixel in the camera plane [9]. The digitized images are compared to match subsets (facets) from one image to another by using an image correlation algorithm. Typically a facet size between 20x20 and 30x30 pixels is chosen. Knowing the imaging parameters for each camera and the orientations of the cameras with respect to each other, the position of each object point in three dimensions can be calculated. If this calculation is done for every point of the object surface, the 3D surface contour of the object can be determined in all areas, which are observed by both cameras. In order to evaluate surface displacements and strains on the object surface, a series of measurements is taken, while the specimen surface is moved due to a loading.

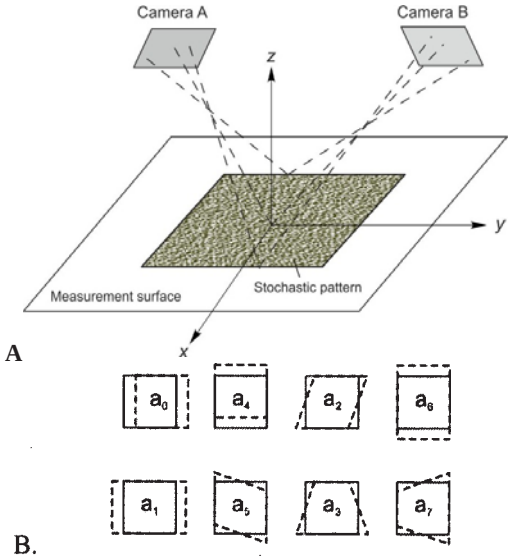


Fig. 1. A. 3D-DIC principle, B. Pseudo-affine transformation

The correlation algorithm tracks the observed gray value patterns for each camera and transforms corresponding facet positions in both cameras into 3D coordinates for each step, resulting in a track of each surface facet in 3D space. As the surface deformation is measured point wise, displacements of individual surface points and subsequently surface strains can be evaluated. The correlation algorithm is based on a pseudo-affine coordinate transformation from one camera image to another:

$$x_t(a_0,a_1,a_2,a_3,x,y) = a_0 + a_1 x + a_2 y + a_3 xy$$
$$y_t(a_4,a_5,a_6,a_7,x,y) = a_4 + a_5 x + a_6 y + a_7 xy \quad (1)$$

The possible transformations consist of a combination of translations, stretch, shear and distortion (Fig. 1B). Within the correlation algorithm the transformation parameters are determined by minimizing the distance between the observed gray value pattern $G_2(x,y)$ in the second image and the original pattern $G_1(x,y)$ by applying the coordinate transformations $(x_t,y_t)$ plus photogrametric corrections, which consider different contrast and intensity levels of the images (illumination parameters $g_0$, $g_1$):

$$G_T(x,y) = g_0 + g_1 G_2\left(x_t(x,y), y_t(x,y)\right) \text{.and}$$
$$R = \min_{a_0,\ldots,a_7,g_0,g_1} \sum \left\| G_1(x,y) - G_T(x,y) \right\| \quad (2)$$

The residuum $R$ is a quality parameter of the correlation algorithm. Such correlation algorithms can determine the maximum of the displacement with an accuracy of up to 1/100 pixel.

This procedure allows the determination of the object deformation in a plane parallel to the image plane of the camera. With the known displacement vectors of each surface point and the reference contour, the strains can be calculated. They can be derived by the analysis of the distortion of each local facet, which has been used for correlation [9].

Before any measurement in case of 3D-DIC a system calibration is necessary to be performed. Its purpose is the determination of the imaging parameters of each of the cameras (intrinsic parameters - focal length, principal point and radial and tangential distortions of the lenses) as well as the external positions and orientations of the cameras with respect to a global coordinate system (extrinsic parameters - translation vector and rotation matrix). The system calibration is needed for transforming image positions on the CCDs of the two cameras of a specimen surface point to the corresponding 3D coordinates of that point. Calibration errors are potentially a major source of systematic evaluation errors, limiting the resolution of the results. The measurement system used in this research (Dantec Dynamics Q400) has, for a successful measurement, a calibration procedure incorporated in the measurement and analysis software. A test plate with a chess model on it is moved in front of the cameras. The software automatically registers the nodal points of the test plate and calculates the intrinsic and extrinsic parameters.

The method of DIC is known to reconstruct displacements with subpixel accuracy and tangential surface strains in the mstrain range. Accuracy of strain measurement using DIC is presented in a large number of publications, but cannot be generalized for a particular experiment. Comparison with other measuring techniques point out that the strain uncertainty measured by DIC [10, 11], are between 100 μm/mm and 400 μm/mm. A general agreement is reached for large deformations (>1%) where DIC is especially suitable. According to the tests performed by the producer of Q400 system [12, 13] the displacement errors are present in the order of less than 0.02 pixels, strain errors are limited to 0.2 mm/m when using a lens with 17 mm focal length. If present displacements are small (lower than 50 pixels), the errors scale linearly. Relative displacement errors are in the order of 0.01-0.05%, strain errors typically count 1-5 με/pixel, related to the existent displacements [12, 13].

In the applications presented here, measured strains are in the range of 100-2000μm/m which is typically accompanied by a large quantity of noise. If the strains are calculated by differentiating the displacement field the numerical differentiation will amplify the noise contained in the calculated displacements. One possibility to deal with this problem is to average the measured strains over a large area where the strain distribution is supposed to be homogeneous. A better result can be obtained calculating in-plane Green-Lagrange strain components by the analysis of the distortion of each local facet as mentioned above [9,12] or simply use linear planes to approximate the computed displacement field [5]

$$u_x(x,y) = A_0 + A_1 x + A_2 y,$$
$$u_y(x,y) = B_0 + B_1 x + B_2 y \quad . \quad (3)$$

where $A_0,\ldots,B_2$ are the desired coefficients, $u_x(x,y)$, $u_y(x,y)$ are the discrete displacements component at coordinates $(x,y)$. The least square method can be applied to determine the $A_0,\ldots,B_2$ coefficients. The strain components can be calculated as

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = A_1, \varepsilon_{yy} = \frac{\partial u_y}{\partial y} = B_2. \quad (4)$$

## Measurement of the thermal expansion coefficient by DIC

*CTE of isotropic materials*

Measurement of the CTE relies on two aspects: accuracy measurement of the thermal displacements by DIC and accuracy of the temperature measurement of the specimen. It is essential for the experimentally determining the CTE that the strain field within the sample to be caused only by the thermal loads and no mechanical restraints on the sample, either external or internal, such that no stresses arise in the body during thermal expansion or contraction.

The CTE can be computed when the strain value and temperature change are known. Using the DIC measuring technique, this presumes recording of gray patterns at two different temperatures $T_1$ and $T_2$. Using specific image processing software one can determine the full-field distributions of displacement and strain belonging to $\Delta T$. The CTE is thus given by.

$$\alpha = \frac{\varepsilon}{\Delta T}. \quad (5)$$

Another known aspect that should be taken into account is the temperature dependence of the CTE $\alpha = \alpha(T)$. Although the CTE is a function of temperature for most materials it can be considered a constant in a large temperature range.

*Influence of the rigid body rotation*

During the thermal expansion an arbitrarily small rigid body rotation of the specimen can occur, the displacement components (fig. 2) can be written

$$\begin{aligned} u_x(x,y) &= \Delta T \cdot x \cdot \alpha - y \cdot \Delta\xi \\ u_y(x,y) &= \Delta T \cdot y \cdot \alpha + x \cdot \Delta\xi \end{aligned} \quad (6)$$

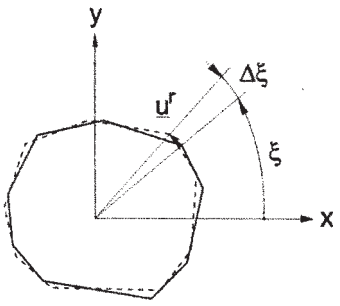


Fig. 2. Influence of a small rigid body rotation

For small deformations, the rigid body rotation angle $\Delta\xi$ can be calculated with the known formula

$$\Delta\xi = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right). \quad (7)$$

The rigid body motions are assumed to be small and they can be easily removed from the DIC procedure by subtracting the displacements of the calculation area centre points.

## Experimental part

*Materials and methods*

The measurement of the CTE-tensor by DIC and the validation of the above described theory about the thermal expansion have been performed for a commercial polypropylene (PP) and polyvinylchloride (PVC). The flat specimens were cut out by milling from an office chair (PP) and a windows profile (PVC) of two local producers. Their size was 27 x 27 mm with a thickness of 3 mm (PVC) and 2 mm (PP) respectively.

Accuracy of the method was investigated on a reference flat sample made of aluminium alloy (EN AW 5754) with the same dimensions (27x27mm) and 3 mm thickness. For measurements a random speckle pattern on the surface of the target specimens is necessary. The pattern

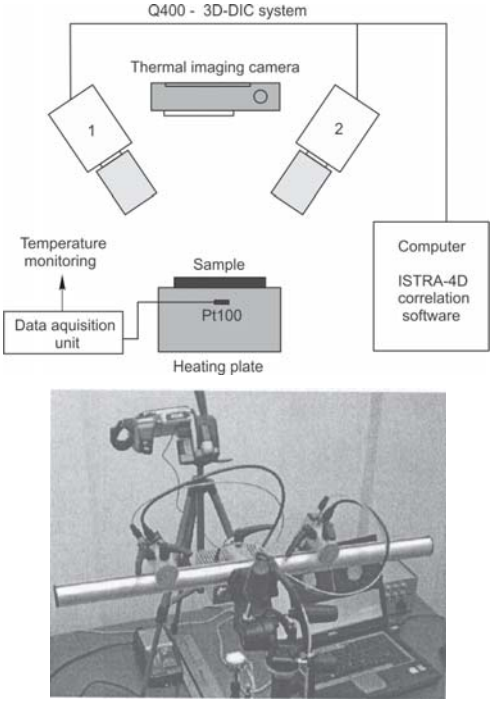


Fig. 3. Experimental set-up for in-plane thermal strain measurement

was produced by black spray applied on an initial painted matt white grounding specimen surface.

Figure 3 shows the experimental set-up developed to measure in-plane thermal strain of the above mentioned specimens. It consists of three main parts: 3D-DIC measuring system, heating device and temperature monitoring (thermal imaging camera and temperature sensors).

Instrumentation for the 3D digital image correlation consist of the Q400 system from Dantec Dynamics (www.dantecdynamics.com) that includes two CCD cameras, 1/8", 1624x1234 pixel resolution, frame rate up to 30 Hz, control electronic, lenses with 17 mm focal length and patented cold light system HILIS (High Intensity LED Illumination System) for very homogenous illumination of the specimen. Accurate temperature measurement was performed simultaneously by a resistance thermometer type PT 100 connected with HBM–Spider8 data acquisition system and mounted near the test sample and with the thermal imaging camera (FLIR, model T400) for monitoring the temperature value and distribution on the top surface of the specimens.

Heating of the test samples was achieved with a temperature-controlled soldering station having attached a massive cupper cylinder and operating between room temperature and 200°C.

In-plane deformations due to thermal expansion at different temperatures were measured by 3D-DIC. Measurements were carried out during heating-up periods, in the interval 20 to 120°C for aluminium and 20°C to 90°C for plastics (PP & PVC) with a temperature steps of 10°C. Temperature at the measurement step was recorded both with a thermo-resistance and the thermal imaging camera.

Uniform heating of the specimen and precise determination of its temperature at the measurement time proved to be a critical point in connection with computation accuracy of the CTE. There are certain parameters which have to be taken into consideration, such as thermal conductivity and the heat transfer from the heating device to the specimen. Several tests with a thermal imaging system demonstrated that a uniform temperature field can be assumed in the test samples during heating periods (fig. 4A). Even for thicker specimens there is a small temperature difference between the bottom surface of the specimen and the top surface of the specimen the temperature difference $\Delta T$ is not affected. The effect can be easily removed by keeping the specimen a certain time (~1 min) at constant temperature but the probability that relative movements between the measuring system and sample get also higher. This temperature difference between specimen faces can cause an out-of-plane bending effect that introduces errors in correct evaluation of the in-plane thermal strain field in case of 2D DIC measurement system. The advantage of 3D-DIC is computation of the specimen contours and considering it for accurate strain calculation. Thermal behaviour of complex polymeric parts used in the industry is particularly suitable to be investigated by 3D-DIC. In this study the

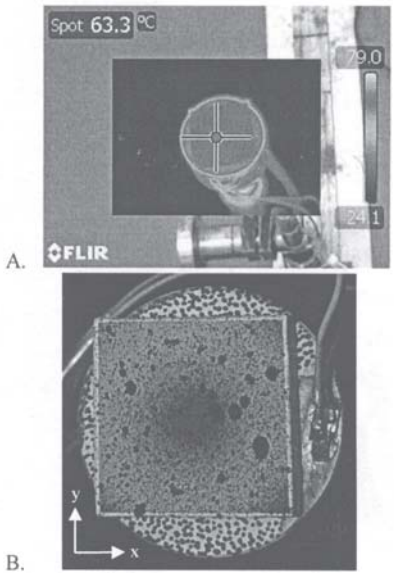


Fig. 4. Thermal behaviour of aluminium sample: A. Temperature distribution acquired with the thermal imaging camera, B. Thermal displacement field

temperature indicated by the thermal imaging camera was used for calculation of temperature difference, $\Delta T$.

From the software adjustable parameters of a DIC system, the most important one affecting the result accuracy is the size of the subsets [14]. Based on previous experience and tests an evaluation grid of 12 pixels with a facet size of 19 pixels and a bi-cubic spline interpolation of the gray values were chosen for evaluation.

As expected, in the ideal case of no rigid body rotations, the isolines of x displacements are parallel with y-axis and those corresponding to y displacements with x-axis. Figure 4B shows the total displacement field of one measuring step with removed rigid body rotation. In the background the perspective view of one camera was superimposed. Most of the measurements were affected by a small specimen rotation. These rigid body rotations are assumed to be small, but, in any case, they are removed from the DIC procedure when calculated the strains from the measured displacements field.

**Results and discussions**

As mentioned in the introduction measurement of small displacements using DIC are accompanied by a significant quantity of noise and implicit a decreased accuracy related the calculated strain.

One possibility to overcome noise influence in calculating the thermal strains is to average strains (derived by the analysis of the distortion of each local facet) over a large area or to use a simplified procedure consisting of linear fitting of the measured displacements as presented

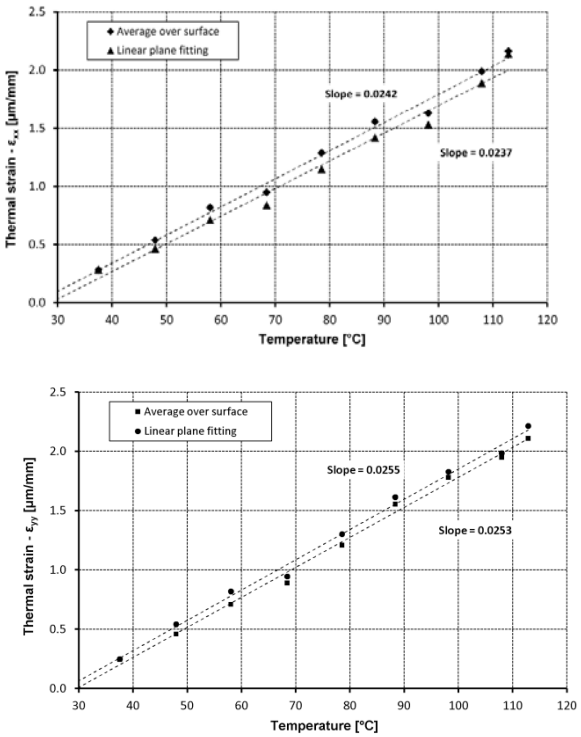


Fig. 5. Variation with temperature of the thermal strains calculated by average over the surface and a linear plane fitting

in [5]. Both procedures to get the thermal strains were compared on an aluminium test sample and presented in figure 5. Analysed area was the entire specimen surface as can be noticed analyzing the figure 4B.

Comparing the two thermal strain calculation methods, in case of aluminium sample, the average relative deviation was 9% for x-direction and 7% for y-direction, that is a good agreement. Both methods can be successfully used. A small advantage has strain average over the area because the measuring software have a rigid body removal option an also a so called "gauge" function to get values over a defined polygonal area. The method proposed by Bing [5] requiring manual rigid body rotation removal and successive manual data fitting, being influenced by the plane position. The average of the slopes of the fitting lines is 23.95 x $10^{-6}$ /°C in the x-direction and 25.4 x $10^{-6}$ /°C in the y-direction, respectively. By comparison with the existed recommended data [15] of 23.7 x $10^{-6}$/°C (25x$10^{-6}$/°C at 100°C [16]) it is proven that the method of 3D-DIC to get the thermal strain is valid and can be successfully applied on materials undergoing large thermal expansion such as the most of polymeric materials.

The displacement of the aluminium sample corresponding to one measurement step ($\Delta T$=10°C) was

**Table 1**
MEASURED CTE VALUES OF POLYMERIC MATERIALS

| Polymeric material | CTE – x direction [x10$^{-6}$ /°C] | CTE – y direction [x10$^{-6}$ /°C] | CTE - average [x10$^{-6}$ /°C] |
|---|---|---|---|
| Polypropylene (PP) | 136.8 | 138.1 | 137.5 |
| Polyvinylchloride (PVC) | 73.7 | 75.3 | 74.5 |

about 7.2 μm but the uncertainty of the measured values decrease from 11% for the first step related to the reference step to 1.1% for the last step with respect to the same reference step. This confirms that for small displacements the accuracy of DIC system is lower but, for materials undergoing large thermal expansion the necessary measurement confidence can be obtained. Measured thermal strains corresponding to a temperature difference of 10°C was about 2.20 μm/mm the strain calculation uncertainty according to the algorithm implemented in the measurement software [12,13] varying from almost 20% in case of the first step with respect to the reference step to 1.3% for the last step corresponding to a temperature difference $\Delta T$=90°C.

It is observed that CTE values in x and y directions are close and also the calculation method by linear plane fitting methods agreed with the average over the area of interest. Both methods can be applied for thermal strain computation, average over the area being easier to be applied in the case of the commercial Dantec Dynamics Q400 measurement system due to its software implementation and reliable strain calculation algorithm. Measured CTE data of polymeric materials are presented in table 1. The results are in good agreement with the reported CTE by different producers or literature: for polypropylene (100...150x10$^{-6}$/°C) and polyvinylchloride (70...80 x10$^{-6}$ /°C).

## Conclusions

In this paper it has been shown that the full-field and real time technique of 3D-DIC is suited to analyse the thermal expansion behaviour and to determine the coefficients of thermal expansion (CTE) for polymeric materials. The method is recommended to be applied for materials that undergo large thermal expansion, such as polymeric materials. For these materials the precision of CTE determination corresponding to a temperature variation of 70°C is about 2x10$^{-6}$/°C. A first condition for such a high accuracy is the precise measurement of the thermal strain. The paper presented a comparative determination of thermal strain, one based on averaging over the area of interest of the in-plane Green-Lagrange strain components and another by a linear plane fitting of the displacement field. Accuracy of the above strain calculation methods was investigated on a reference flat sample made of aluminium alloy and revealed that both methods can be successfully applied. The second condition for a reliable CTE determination is the precise temperature measurement and control. The temperature monitoring set-up performed simultaneously by a resistance thermometer connected at a data acquisition system and mounted near the test sample and with a thermal imaging camera satisfied the high requirements of temperature measurement of the sample.

Even the accuracy is lower than that measured by interferometric techniques [2], the DIC enjoys besides of advantages being non-contact and full-field measurement method, to have a simple set-up, low sensibility to environmental conditions, easy post-processing of the measured data, no limits on the temperatures and strains than can be reached [8].

Validating the application of a commercial 3D-DIC measurement system to CTE determination proved to be a good start point to future determination in situ of the thermal behaviour of complex polymeric parts used in the industry.

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