Static and Dynamic Young's Modulus Determination of a Bone Substituent

ALEXANDER BABIA, IOAN PARAUSANU, HORIA-MIRON GHEORGHIU*
Politehnica University of Bucharest, 313 Splaiul Independentei, 060042, Bucharest, Romania

Young's modulus needed for realizing a numerical analysis model with finite elements dedicated to studying static strain on an artificial bone, made out of composite material (polyester resin reinforced with fiber glass) is determined, as well as Young's dynamics modulus needed for analysis through numerical simulation of dynamic strain applied to the same structural element (vibrations, shock). Both in, static and dynamic conditions, two methods are utilized in measuring the materials elastic characteristics.

Keywords: Young's modulus, artificial bone, composite material, numerical simulation

An important field in biomechanical studies is the analysis of the bones behaviour when under external forces, in static or dynamic conditions. In order to use the similarity in experimental findings or in the method of finite elements in computerized simulations, mechanical characteristics of both the bone and the substitute material are necessary. Evidently, for studies of general nature, the values for different material categories provided by the field literature can be used [8]. This solution is satisfactory in the case of alloys, for example, which have elastic characteristics that present relatively small variations in comparison to chemical composition variations. For other material categories [7], which do not have elastic characteristics that depend only on chemical composition, but also on the development technology, such as polymer materials or polymer matrix composites, to ensure the accuracy of the results obtained after numerical analysis, these characteristics need to be experimentally obtained on specimens made-up from the structural element analyzed [2, 5].

In principle, such an experiment consists in subjecting the specimen to a force of variable intensity and measuring the resulting deformations. The link established between the force and the deformations, based one Hooke's law allows determining Young's modulus [3].

The protocol for conducting the experiment provides conditions for strain growth. For experiments in which strains do not vary in time, the Young's static modulus used is determined with a small enough growth speed.

Young's dynamic modulus is used in determining strains which vary in time (vibrations, shocks) [1, 4].

Using materials to substitute bone is frequent today. Usually, they are used to manufacture different types of implants or prostheses. Such materials can also be used to validate the calculation model for different studies linked to bone structure behaviour, in different strain conditions, thus, avoiding costly, in vivo or in vitro experiments that use real bone as material. Due to this necessity, the following will show trial experiments made to determine Young's modulus, used in the studies of the elastic strains, under static or dynamic loads.

Experimental part

Material and specimens

A composite material with a polyester resin matrix reinforced with fiber glass chips was used for the findings.

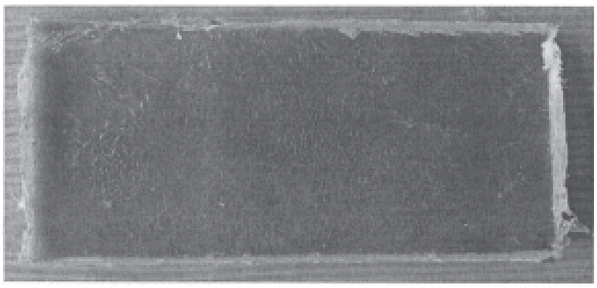

Fig 1. Composite material, polyester resin reinforced with fibers glass (chips)

The material is made of several layer fibers glass board with random distribution (fig. 1), soaked in resin, prepared in accordance with the manufacturer's directions regarding resin-hardener proportions, temperature and curing time.

From the resulting board, considered to have transverse isotropy, specimens were made for stretching trials (fig 2), bending trials "in three points" and dynamic trials (fig 3.). In table 1 are the specimen dimensions.

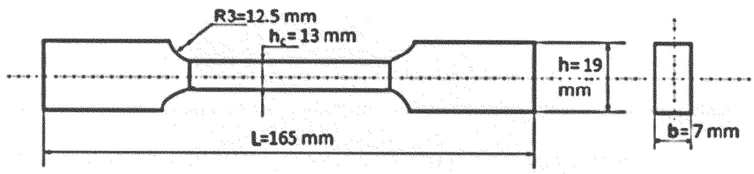

Fig 2, Sample for tensile test

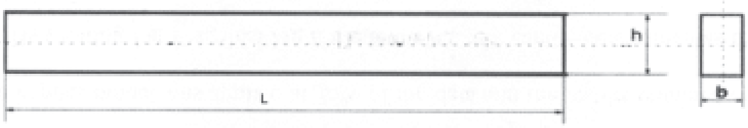

Fig.3. Sample for three points flexion and vibration tests

* email: hgheorghiu@yahoo.com

Trail	L [mm]	b [mm]	h [mm]
Three points bending	165	19	7
Dynamic continuous body model	430 ± 5	14 ± 1	2
Dynamic body model with one degree of freedom	250	20	6

Table 1
SPECIMEN DIMENSIONS

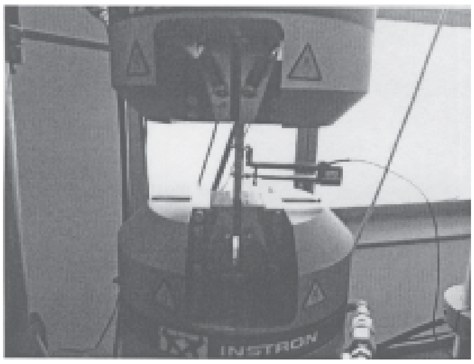

Fig 4. Traction trial

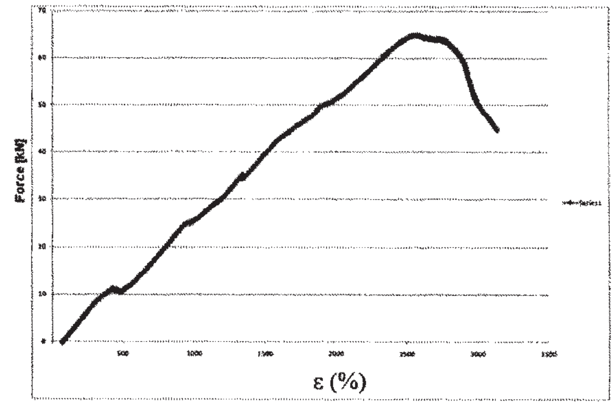

Fig 5. The specimen's characteristic curve

Young's Modulus determination in static stress

From the relation between the applied force and the displacement it creates, the Young's modulus was determined. Two types of tests were performed: traction and bending.

Traction trials

The trials were conducted using an INSTRON 8800 machine, in the Strength of Materials Department laboratory within the Polytechnic University of Bucharest. The machine provides the characteristic curve of the material tested (fig 4), both in graphic and tabular form.

To comply a static strain, the movement growth rate was established at 0.5 mm/min, considered to be low enough to not influence the trial results. The experiments were conducted in a normal temperature and humidity environment.

The traction trials resulted in a characteristic curve for each of the specimen used, are exemplified in figure 5.

Analyzing this test, the existence of a quasi linear sector is observed, whose slope represents Young's Modulus, given by the relation

$$E_i = \frac{(\Delta F)_i}{A \cdot (\Delta \varepsilon_l)_i} \cdot 10^6 , \ [MPa] \qquad (1)$$

where $(\Delta F)_i$ represents a range of variation of the force applied [N], in the linear sector of the characteristic bend, and $(\Delta \varepsilon_l)_i$ represents the specific longitudinal strain variations corresponding to the forces at the ends of this range. (A) represents the surface of the specimen's transverse section [m^2].

Young's static strain modulus was obtained by mediating the results.

In table 2 are the elasticity modulus's values obtained for the 12 specimens, which were used to calculate the average E = 70387 GPa.

Bending trials

The trial is accomplished by measuring the maximum deflection, f_{max}, produced by a known force F, applied to the middle of a double supported beam with the opening of l00 mm, such as in figure 6.

Using the maximum deflection expression for this force,

$$f_{max} = \frac{Fl^3}{48EI_y} \qquad (2)$$

the Young's modulus the relation ensues

$$E = \frac{Fl^3}{48 f_{max} I_y} \qquad (3)$$

Sample.	Young's Modulus, E[MPa]
1	70320
2	70327
3	70481
4	70450
5	70060
6	70490
7	70501
8	70385
9	70700
10	70123
11	70390
12	70421
Average value	70387

Table 2
YOUNG'S MODULUS FOR THE SUBSTITUTE MATERIAL DETERMINED BY TRACTION TRAIL

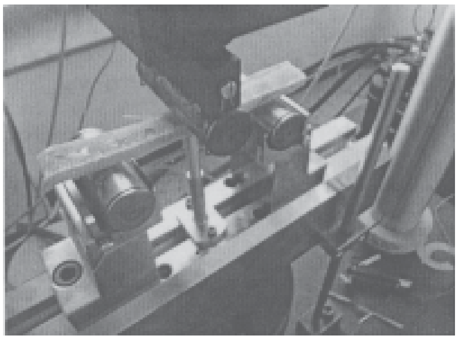

Fig. 6 Three points bending trail

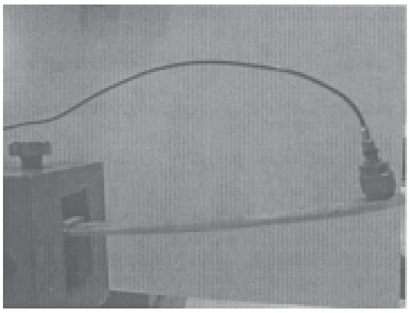

Fig. 7 Piezoelectric accelerometer fixed on the embedded rod

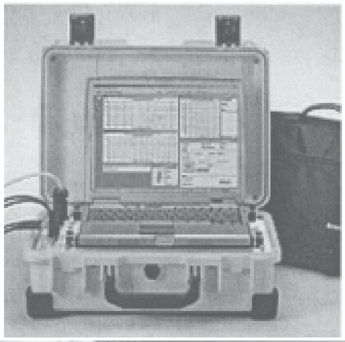

Fig. 8. Brüel & Kjćr PULSE measuring system

Table 3
YOUNG'S MODULUS FOR THE SUBSTITUTE MATERIAL DETERMINED BY THREE POINTS BENDING TRAIL

Sample	Young's modulus, E [MPa]
1	89991
2	92403
3	88791
4	89815
5	88002
6	90075
7	89870
8	90462
9	90314
10	90008
11	90001
12	90314
Average value	90004

Trials conducted on the INSTRON 8800 for 12 specimens led to the results portrayed in table 3.

Determination of Young's dynamic modulus

The dependency of the mechanical characteristics of the load speed grows shows that for time dependent phenomena's (vibrations, sock, fatigue) analysis, Young's dynamic modulus must be used. Due to the fact that literature does not provide enough specific data regarding this parameter values, it was experimentally determined through vibration trials conducted within the Vibrations Laboratory of the Strength of Materials Department from the Polytechnic University of Bucharest.

Rectangular shaped rod specimens were embedded at one of the ends and left to vibrate freely as a result to an initial applied impulse.

The signal transmitted by a 9g piezoelectric accelerometer placed at the free end was used to record the vibratory motion. (fig. 7). A Brüel & Kjœr PULSE measuring system (fig. 8), suitable for vibration recording and analysis, was used to process the data transmitted by the accelerometer.

From the amplitude-frequency spectrum provided by the measuring system, the natural frequencies are obtained

$$f_p = \frac{\omega_n}{2\pi}, \tag{4}$$

where ω_n is the pulsation corresponding to the amplitudes' peek value within the recorded spectrum.

Young's dynamic modulus values were determinated by using two models in which the tested specimen was treated as an elastic element [6]:

a) the aim of the first case was the dynamic behavior of the embedded bar with a concentrated mass m = 0.2kg at the free extremity (fig 9), considered as body with a single degree of freedom (fig. 10) and whose calculation model is shown in figure 11.

The natural frequencies of the model in figure 10 was experimentally obtained using the answer spectrum in frequency registered during free vibration. In figure 12, such a spectrum is shown for one of the four tested specimens.

From the relation of the natural frequency,

$$p = \sqrt{\frac{k}{m}}, \tag{5}$$

where $k = 3EI / l^3$, k - the elastic constant of the embedded rod.

The resulted relation for the Young's modulus is

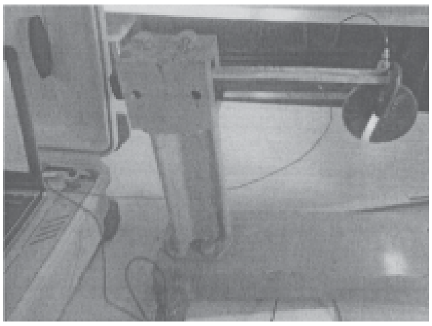

Fig. 9. Dynamic trail for the material substitut

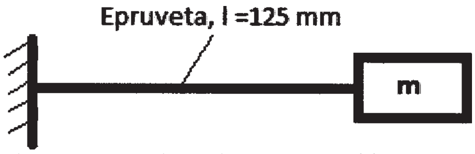

Fig. 10. Model with a single degree of freedom

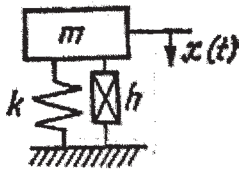

Fig. 11. Calculation model

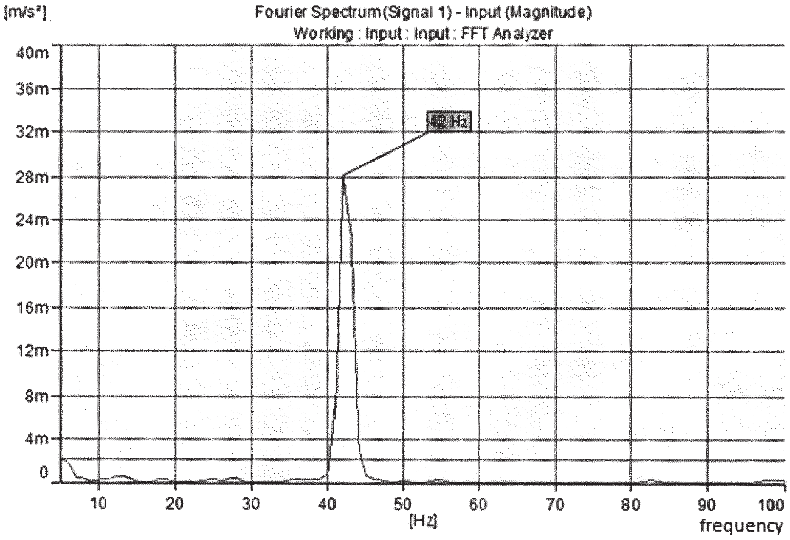

Fig. 12 Spectrum in frequency, single degree of freedom model

Table 4

Probe	Mass [Kg]	Length [m]	Section cross [m]		Natural frequency [Hz]	Young's dynamics modulus E [MPa]
			h	b		
1	0.2	0.25	0.006	0.021	29	**90385**
2	0.2	0.25	0.006	0.020	28.5	**91660**
3	0.2	0.25	0.006	0.015	27	**91406**
4	0.2	0.245	0.006	0.022	29	**81203**
Average value						**88663**

Fig. 13 Specimen sample shape

$$E = \frac{p^2 m l^3}{3I}, \qquad (6)$$

with I – the quadratic moment of the cross-section, p - natural pulsation, f - the natural frequency of the vibration at the amplitudes peek value, read on the recorded spectrum.

b) In the second case, the rod embedded at one end was considered as a continuous body.

Specimens, with the shape as shown in figure 13, were embedded at one end. The mass of the accelerometer set on the bar was considered negligible (fig. 14). Removed from the equilibrium position, the bar carries out a continuous body movement.

Fig. 14 Specimen embedded at one end

The natural pulsations equation in this case is

$$ch(\propto l) \times \cos(\propto l) = 1 \qquad (7)$$

whose roots are $(\propto l)_1 = 4.73$, $(\propto l)_2 = 7.85$, $(\propto l)_3 = 10.99$, ..., where l is lengths of the bare and

$$\propto^4 = \omega_p^2 \frac{\rho A}{E I}, \qquad (8)$$

w_p – natural pulsation, ρ - the material density, A - specimen section surface, E - elastic modulus and I - axial inertia moment of the section.

From this reference results

$$\omega_p = \alpha^2 \sqrt{\frac{E\,I}{\rho A}} = \frac{(\alpha\, l)^2}{l^2} \sqrt{\frac{E\,I}{\rho A}}, \qquad (9)$$

from which the naturals frequency expression is obtained

$$f = \frac{\omega_p}{2\pi} = \frac{(\alpha\, l)^2}{2\pi l^2} \sqrt{\frac{E\,I}{\rho A}}, \qquad (10)$$

Considering for f the value of the first natural frequency experimentally determined using the spectrum in frequency (fig. 15) and for αl the first root of the equation (7), it results:

$$E = \frac{4\pi \rho A l^4 f^2}{(\alpha l)_1^4\, I}. \qquad (11)$$

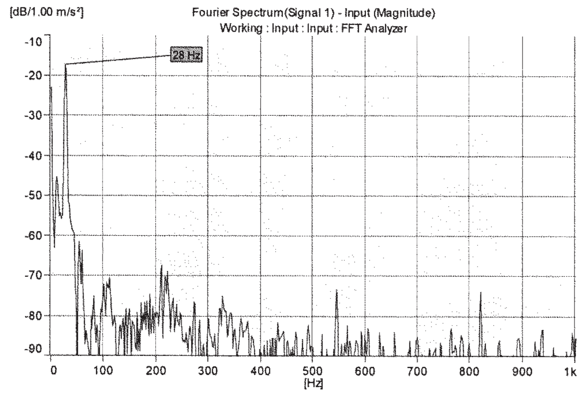

Fig 15 Spectrum in frequency, continous body

Table 5

Sample	Mass [Kg]	Length [m]	Cross section [m]		Natural frequency [Hz]	Young's dynamic modulus E [MPa]
			h	b		
1	0.08	0.43	0.002	0.015	42	**87420**
2	0.075	0.43	0.002	0.014	42.5	**89914**
3	0.08	0.435	0.002	0.015	43	**94866**
4	0.07	0.43	0.002	0.013	42.8	**91655**
Average value						**90963**

For the four specimens, the obtained dynamic elastic modulus's values are found in table 5.

Conclusions

For the static trails, the values determined using the two methods for Young's static modulus, stretching or bending, are relatively close.

The differences can be assigned to the way in which the material was crafted, but small enough so that the differences can be considered insignificant.

For dynamic stress analysis, the use of Young's dynamic modulus is important due to the fact that the values obtained are different that the ones obtained through static trials. For this material, the results show that E dynamic is different with approximately 20% that the static one.

References

1. ALAN, W., Dynamic Elastic Modulus Measurements in Materials, ASTM International, 1990

2. *** American Society of Civil Engineers , Design of Fiberglass-Reinforced Plastic (Frp) Stack, ASCE Publications, 2010.

3. DOWLING, N. E., Mechanical behavior of materials engineering methods for deformation, fracture and fatigue (3rd Ed.),Lavoisier S.A.S, 2009

4. HORST, C., TETSUYA, S., LESLIE, S., Springer Handbook of Metrology and Testing, Springer, 2011

5. MANAS, C., SALIL, K. R., Plastics Fundamentals, Properties, and Testing, CRC Press, 2010

6. RADES, M., Modele mecanice pentru identificarea sistemelor dinamice, Editura Academiei 1969, Bucuresti

7. VISHU, S., Handbook of Plastics Testing and Failure Analysis, John Wiley & Sons, 2007

8. *** Matweb, www.matweb.com

Manuscript received: 11.02.2014